Astronomy

before the Telescope

Edited by Christopher Walker

With a foreword by Patrick Moore

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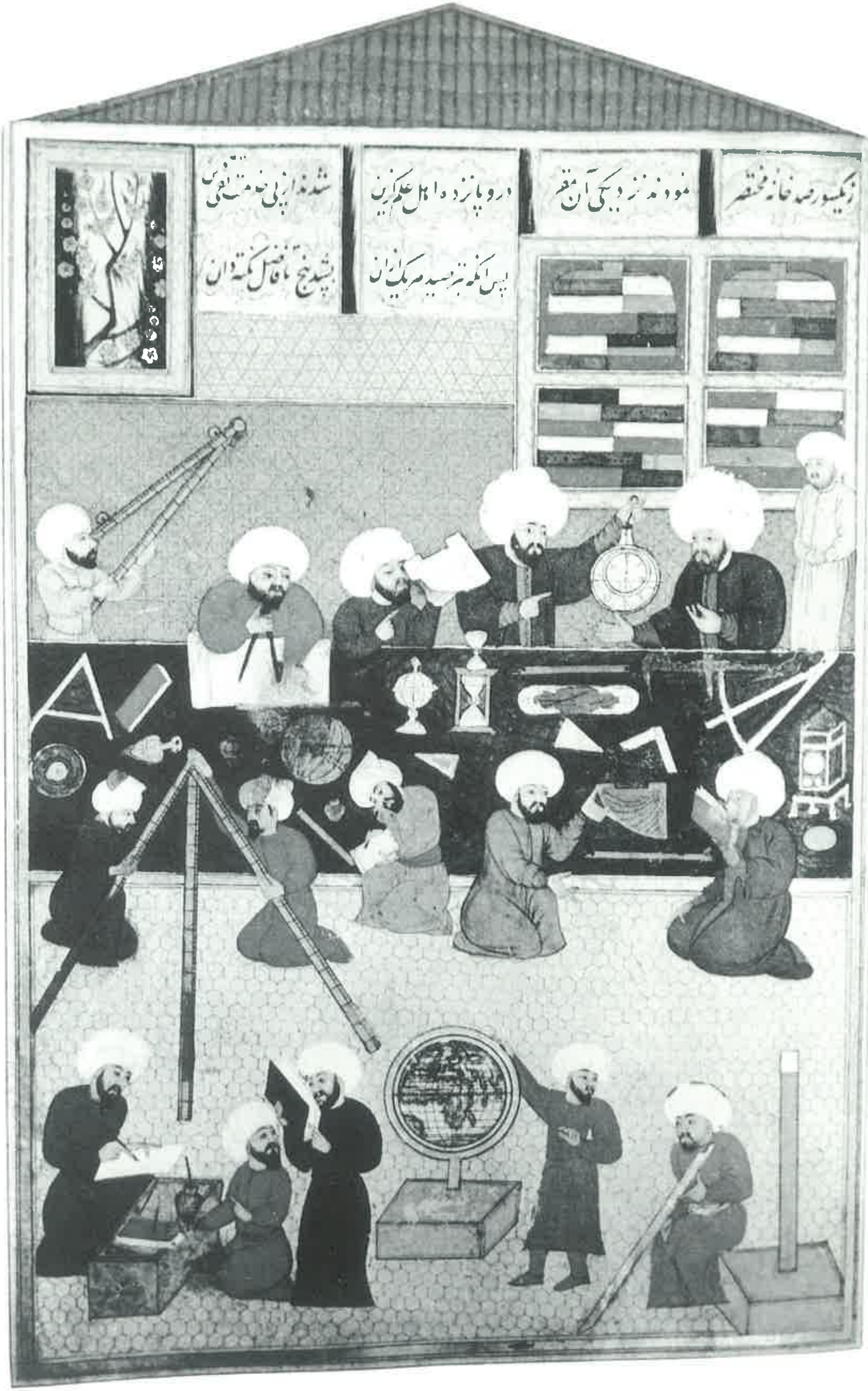
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Frontispiece The Istanbul observatory around the year 1577. The director of the observatory, Taqi l-Dīn, is holding an astrolabe; the other astronomers are involved with various instruments, of which only the terrestrial globe and the mechanical clock are European imports. See further chapter 9. (University Library, Istanbul, MS Yıldız 1404)

Contents

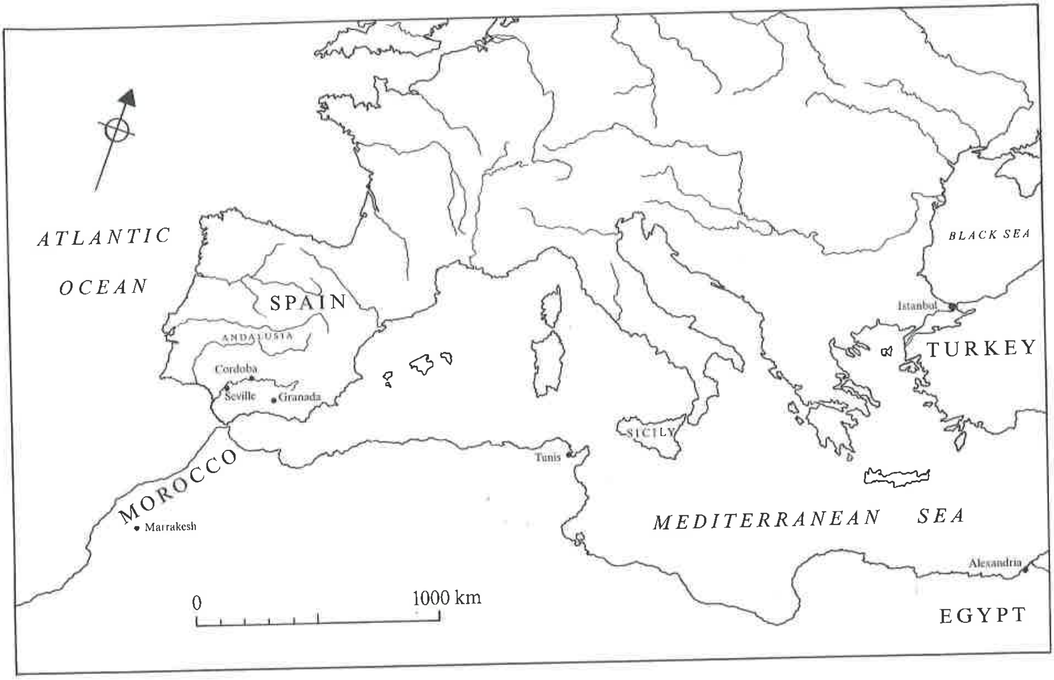
DAVID A. KING

Islamic Astronomy

The nature of Islamic astronomy

From the ninth century to the fifteenth, Muslim scholars excelled in every branch of scientific knowledge. In particular their contributions to astronomy and mathematics are impressive. There are an estimated 10,000 Islamic astronomical manuscripts and close to 1,000 Islamic astronomical instruments preserved in libraries and museums in the Near East, Europe and North America, but it is clear that even if all of them were properly catalogued and indexed – and we are still very far from this state of affairs – the picture that we could reconstruct of Islamic astronomy, especially for the eighth, ninth and tenth centuries, would be quite deficient. Most of the available manuscripts and instruments date from the later period of Islamic astronomy, that is, from the fifteenth to the nineteenth century, and although some of these are based or modelled on earlier works many of the early works are extant in unique copies and others have been lost almost without trace, that is, we sometimes know only of their titles. The thirteenth-century Syrian scientific biographer Ibn al-Qifṭī relates that the eleventh-century Egyptian astronomer Ibn al-Sanbadī heard that the manuscripts in the library in Cairo were being catalogued and so he went to have a look at the works relating to his field. He found 6,500 manuscripts relating to astronomy, mathematics and philosophy. Not one of these survives amongst the 2,500 scientific manuscripts preserved in Cairo today.

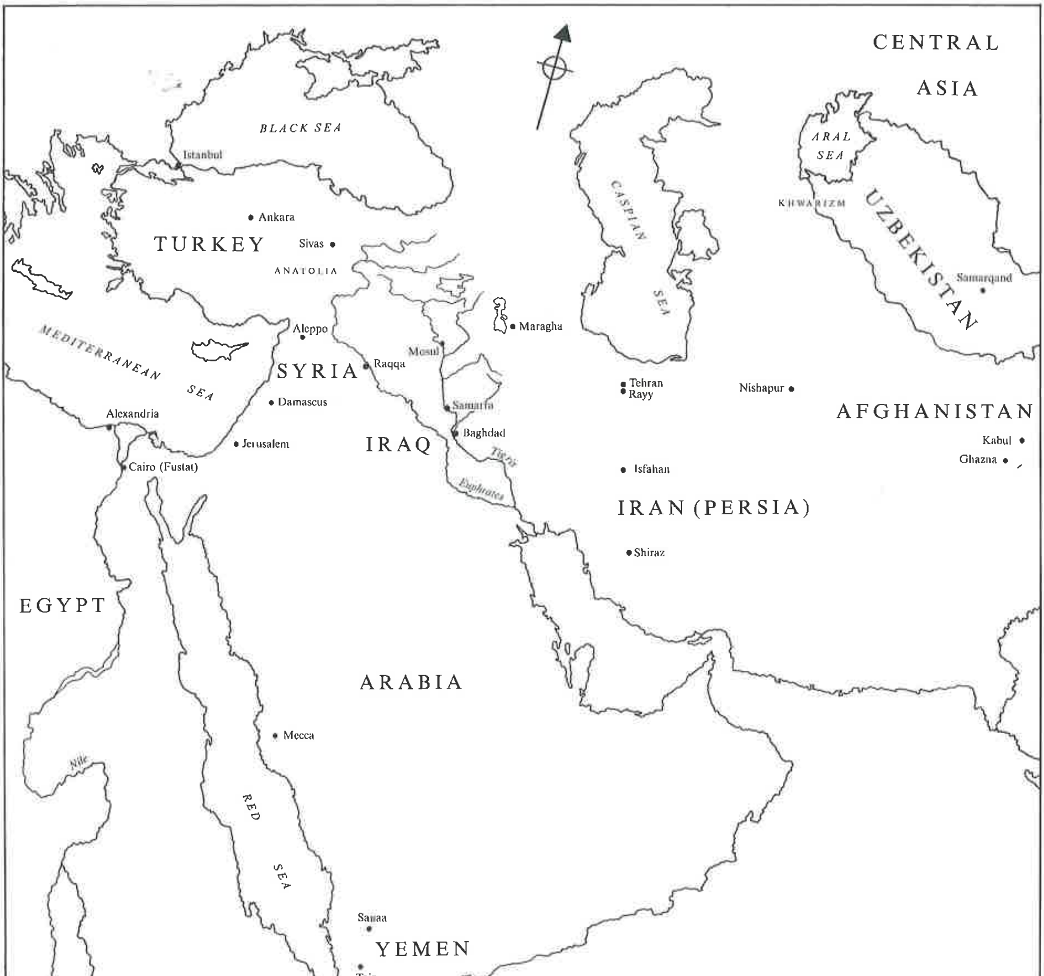
The surviving manuscripts thus constitute but a small fraction of those that were actually copied; nevertheless they preserve for us a substantial part of the Islamic scientific heritage, certainly enough of it for us to judge its level of sophistication. Only in the past few decades has the scope of the activity and achievements of Muslim scientists become apparent, and the days are long past when they were regarded merely as transmitters of superior ancient knowledge to ignorant but eager Europeans. Islamic astronomy is to be viewed on its own terms. The fact that only a small part of the available material, mainly Greek and Indian material in Arabic garb, was indeed transmitted to Europe is to be viewed as an accident of Islamic history. There is no need to apologise for using the expression 'Islamic astronomy'. Within a few decades of the death of the



47 (*above and opposite*) The Islamic world.

Prophet Muḥammad in 632 the Muslims had established a commonwealth stretching from Spain to Central Asia and India. They brought with them their own folk astronomy, which was then mingled with local traditions, and they discovered the mathematical traditions of the Indians, Persians and Greeks, which they mastered and adapted to their needs. Early Islamic astronomy was thus a pot-pourri of pre-Islamic Arabian star-lore and Indian, Persian and Hellenistic astronomy, but by the tenth century Islamic astronomy had acquired very distinctive characteristics of its own. A. I. Sabra labels this process ‘appropriation and naturalisation’.

We should point out at the outset that astronomy flourished in Islamic society on two different levels: folk astronomy, devoid of theory and based solely on what one can see in the sky, and mathematical astronomy, involving systematic observations and mathematical calculations and predictions. Folk astronomy was favoured by the scholars of the sacred law (*fuqahā’*), not least because of various religious obligations that demanded a basic knowledge of the subject; these legal scholars generally had no time (or need) for mathematical astronomy. That discipline was fostered by a select group of scholars, most of whose activities and pronouncements were, except in the case of astrological predictions, of little interest to society at large.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Damascus
Samarra
Baghdad
Tigris
Euphrates
Alexandria
Jerusalem
Cairo (Fustat)
IRAQ
Tehran
Rayy
Nishapur
AFGHANISTAN
Kabul
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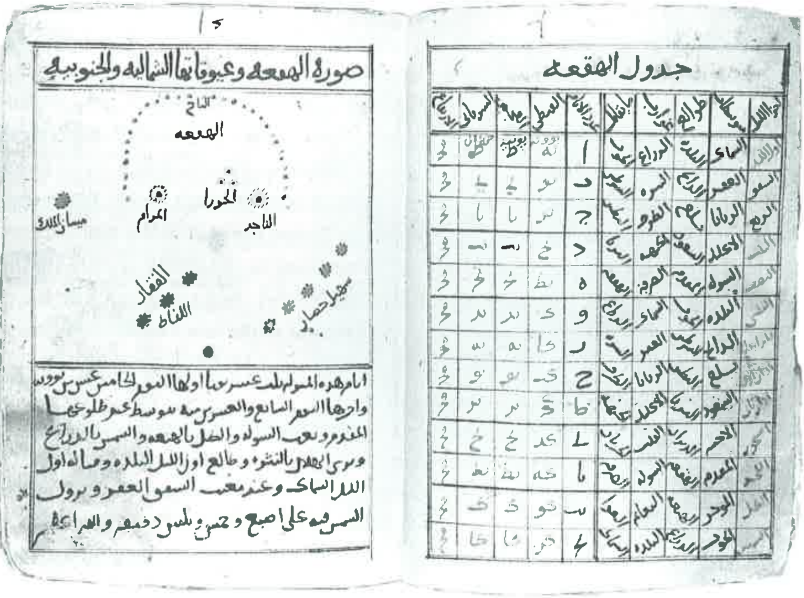
As we shall see, the astronomers also played their part in applying their discipline to certain aspects of Islamic religious practice. It was not Islam that encouraged the development of astronomy but the richness of Islamic society, a multiracial, highly-literate, tolerant society with one predominant cultural language, Arabic. But neither did Islam, the religion, stand in the way of scientific progress. The Prophet had said: 'Seek knowledge, even as far as China'. To be sure, over-zealous orthodox rulers occasionally pursued, killed or otherwise attacked 'scientists' or destroyed or burnt their libraries, but these were exceptions. The scholars of the religious law, who saw themselves as the representatives of Islam, generally ignored the pronouncements of the scientists, even on matters relating to religious practice. Astronomy was the most important of the Islamic sciences, as we can judge by the volume of the associated textual tradition, but a discussion of it in the broader context of the various branches of knowledge, which has been attempted several times elsewhere, is beyond the scope of this chapter.

Arab star-lore

The Arabs of the Arabian peninsula before Islam possessed a simple yet developed astronomical folklore of a practical nature. This involved a knowledge of the risings and settings of the stars, associated in particular with the cosmical settings of groups of stars and simultaneous heliacal risings of others, which marked the beginning of periods called *nawʾ*, plural *anwāʾ*. These *anwāʾ* eventually became associated with the twenty-eight lunar mansions (Fig. 48), a concept apparently of Indian origin. A knowledge of the passage of the sun through the twelve signs of the zodiac, associated meteorological and agricultural phenomena, the phases of the moon, as well as simple time-reckoning using shadows by day and the lunar mansions by night, formed the basis of later Islamic folk astronomy, which flourished separately from mathematical astronomy in Islamic society.

More than twenty compilations on the pre-Islamic Arabian knowledge of celestial and meteorological phenomena as found in the earliest Arabic sources of folklore, poetry and literature, are known to have been compiled during the first four centuries of Islam. The best known is that of Ibn Qutayba, written in Baghdad about the year 860. Almanacs enumerating agricultural, meteorological and astronomical events of significance to local farmers were also compiled: several examples of these survive from the medieval Islamic period, one such being for Córdoba from the year 961. The Yemen possessed a particularly rich tradition of folk astronomy, and numerous almanacs were compiled there.

Since the sun, moon and stars are mentioned in the Quran, an extensive literature dealing with what may well be labelled Islamic folk cosmology arose. This was inevitably unrelated to the more 'scientific' Islamic tradition based first on Indian sources and then predominantly on Greek ones. Since it is also stated in the Quran that man should use these celestial bodies to guide him, the scholars of the religious law occupied themselves with folk astronomy. We shall mention below various treatises dealing



48 An illustration of the stars of one of the lunar mansions (*al-haqʿa*) in an Egyptian treatise on folk astronomy. The table on the right identifies the mansions culminating, rising and setting, and the mansions opposite them (at 180°) at different times of the night when the sun is in that mansion; it also gives the date in the Coptic, Western and Syrian calendars and the midday solar altitude when the sun is in each of the 13 degrees of the mansion ($360 \div 28 \approx 13$). The associated text repeats some of this information and adds the midday shadow length. (Chester Beatty Library, Dublin, MS 4538)

with simple time-keeping and the determination of the direction of Mecca by non-mathematical means.

Persian and Indian sources

The earliest astronomical texts in Arabic seem to have been written in Sind and Afghanistan, areas conquered by the Muslims already in the seventh century. Our knowledge of these early works is based entirely on citations from them in later works. They consisted of text and tables and were labelled *zīj* after a Persian word meaning ‘cord’ or ‘thread’ and by extension ‘the warp of a fabric’, which the tables vaguely resemble. The Sasanian *Shahriyārān Zīj* in the version of Yazdigird III (see p. 135) was translated from Pahlavi into Arabic as the *Shāh Zīj*, and the astronomers of the Caliph al-Manṣūr chose an auspicious moment to found his new capital Baghdad using probably an earlier Pahlavi version of this *zīj*. The various horoscopes computed by Māshā’allāh (Baghdad, *c.* 800) in his astrological world history are based on it.

Significant for the subsequent influence of Indian astronomy in the Islamic tradition was the arrival of an embassy sent to the court of al-Manṣūr from Sind *c.* 772. This embassy included an Indian well versed in astronomy and bearing a Sanskrit astronomical text apparently entitled the *Mahāsiddhānta* and based partly on the *Brāhmasphuṭasiddhānta* (see pp. 136–7). The Caliph ordered al-Fazārī to translate this text into Arabic with the help of the Indian. The resulting *Zīj al-Sindhind al-kabīr* was the basis of a series of *zīj*es by such astronomers as al-Fazārī, Yaʿqūb ibn Ṭāriq, al-Khwārizmī, Ḥabash, Ibn Amājūr, al-Nayrīzī, and Ibn al-Ādamī, all prepared in Iraq before the end of the tenth century. The *Sindhind* tradition (see pp. 133 and 137) flourished in Andalusia, mainly through the influence there of the *Zīj* of al-Khwārizmī (see below). As a result, the influence of Indian astronomy is attested from Morocco to England in the late Middle Ages.

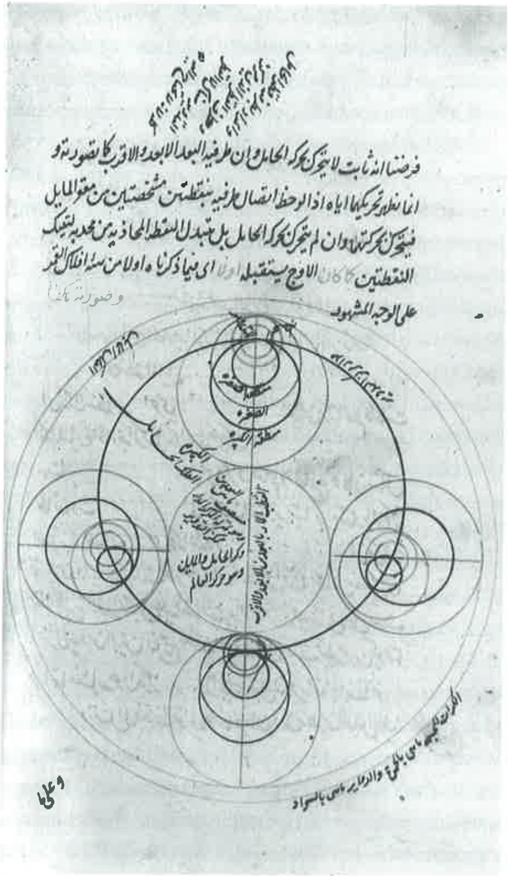
Greek sources

Ptolemy's *Almagest* was translated at least five times in the late eighth and ninth centuries. The first was a translation into Syriac and the others were into Arabic, the first two under the Caliph al-Ma'mūn in the middle of the first half of the ninth century, and the other two (the second an improvement of the first) towards the end of that century. All of these were still available in the twelfth century, when they were used by Ibn al-Ṣalāḥ for his critique of Ptolemy's star catalogue. The translations gave rise to a series of commentaries on the whole text or parts of it, many of them critical and one, by Ibn al-Haytham (*c.* 1025), actually entitled 'Doubts about Ptolemy' (*al-Shukūk*). The most commonly used version of the *Almagest* in the later period was the recension of the late ninth-century version by the polymath Naṣīr al-Dīn al-Ṭūsī in the mid thirteenth century. Various other works by Ptolemy, notably the *Planetary Hypotheses* and the *Planisphaerium*, and other Greek works, including the short treatises by Autolycus, Aristarchus, Hypsicles and Theodosius, and works on the construction known as the analemma for reducing problems in three dimensions to a plane, were also translated into Arabic; most of these too were later edited by al-Ṭūsī. In this way Greek planetary models, uranometry and mathematical methods came to the attention of the Muslims. Their redactions of the *Almagest* not only contained reformulations and paraphrases of its contents but they also 'corrected, completed, criticized and brought [the contents] up to date both theoretically and practically' (Saliba); most of this material has not been studied in modern times.

Developments in astronomy

Theoretical astronomy

The geometrical structure of the universe conceived by Muslim astronomers of the early Islamic period (*c.* 800–1050) is more or less that expounded in Ptolemy's *Almagest*, with the system of eight spheres being regarded essentially as mathematical models. However, already in Ptolemy's *Planetary Hypotheses* these models are taken as representing physical



49 A non-Ptolemaic planetary model for the moon, found in a copy of the treatise on planetary astronomy (*al-Tuḥfa al-shāhiyya*) written by Quṭb al-Dīn al-Shīrāzī in Sivas (Anatolia) in AD 1285. Only since the 1950s have these models been investigated by modern scholars; the discovery that a series of Muslim astronomers concerned themselves with such models from the eleventh to the sixteenth centuries and developed models without the problems inherent in the Ptolemaic ones has promoted considerable interest in medieval Islamic planetary theory. (Egyptian National Library, Cairo, MS K3758)

reality; this text also became available in Arabic. Several early Muslim scholars wrote on the sizes and relative distances of the planets, and one who proposed a physical model for the universe was Ibn al-Haytham (fl. *c.* 1025). In order to separate the two motions of the eighth sphere, the motion of the fixed stars due to the precession of the equinoxes and the motion of the fixed stars due to the apparent daily rotation, he proposed a ninth sphere to impress the apparent daily rotation on the others.

Of considerable historical interest are various Arabic treatises on the notion of the trepidation of the equinoxes. This theory, developed from Greek sources, found followers who believed that it corresponded better to the observed phenomena than a simple theory of uniform precession. The mathematical models proposed were complex and have only recently been studied properly (notably those of Pseudo-Thābit (date unknown) and Ibn al-Zarqāllu (Andalusia, *c.* 1070), who seems to have relied on his predecessor Ṣāʿid al-Andalusī). The theory of trepidation continued to occupy certain Muslim scholars (in the late period mainly in the Maghrib), as it did European scholars well into the Renaissance. The history of this notion has yet to be written.

Other significant Islamic modifications to Ptolemaic planetary models, devised to overcome the philosophical objections to the notion of an equant and the problem of the variation in lunar distance inherent in Ptolemy's lunar model, belong to the later period of Islamic astronomy (Fig. 49). There were two main schools, one of which reached its fullest expression in Maragha in North-west Iran in the thirteenth century (notably with al-Ṭūsī and his colleagues) and Damascus in the fourteenth (with Ibn al-Shāṭir), and the other of which developed in Andalusia in the late twelfth century (notably with al-Biṭrūjī). The latter tradition was doomed from the outset by a slavish adherence to (false) Aristotelian tenets and mathematical incompetence. The former was based on sophisticated modifications to Ptolemy's models, partly inspired by new observations; Ptolemy himself would have been impressed by it, as have been modern investigators, for the tradition has been rediscovered and studied only in the latter half of this century. In the 1950s E. S. Kennedy discovered that the solar, lunar and planetary models proposed by Ibn al-Shāṭir in his book *The Final Quest Concerning the Rectification of Principles* (*Nihāyat al-su'l*) were different from those of Ptolemy, indeed that they were mathematically identical to those of Copernicus some 150 years later. In this work Ibn al-Shāṭir 'laid down the details of what he considered to be a true theoretical formulation of a set of planetary models describing planetary motions, and actually intended as alternatives to the Ptolemaic models' (Saliba). He maintained the geocentric system, whereas Copernicus proposed a hypothesis, which he was unable to prove, that the sun was at the centre of things. Nevertheless this important discovery raised the interesting question whether Copernicus might have known of the works of the Damascene astronomer (see p. 202). Since the 1950s we have progressed to a new stage of inquiry: we now know that there was a succession of Muslim astronomers from the eleventh century to the sixteenth who concerned themselves with models different from those of Ptolemy, all designed to overcome what were seen as flaws in them. The question we may now ask is: was Copernicus influenced by any of these Muslim works? The answer is unsatisfactory, namely, that he must have been; definitive proof is, however, still lacking.

Mathematical astronomy – the tradition of the zījes

The Islamic *zīj*es constitute an important category of astronomical literature for the historian of science, by virtue of the diversity of the topics dealt with, and the information that can be obtained from the tables. In 1956 E.S. Kennedy published a survey of about 125 Islamic *zīj*es. We now know of close to 200, and material is available for a revised version of the *zīj* survey. To be sure, many of these works are lost, and many of the extant ones are derived from other *zīj*es by modification, borrowing, or outright plagiarism. Nevertheless, there are enough *zīj*es available in manuscript form to reconstruct a reasonably accurate picture of the Islamic activity in this field.

Most *zīj*es consist of several hundred pages of text and tables; the treatment of the material presented may vary considerably from one *zīj* to another. The following aspects of mathematical astronomy are handled in a typical *zīj*:

1) chronology
2) trigonometry
3) spherical astronomy
4) solar, lunar and planetary mean motions
5) solar, lunar and planetary equations
6) lunar and planetary latitudes
7) planetary stations
8) parallax
9) solar and lunar eclipses
10) lunar and planetary visibility
11) mathematical geography (lists of cities with geographical co-ordinates), determination of the direction of Mecca
12) uranometry (tables of fixed stars with co-ordinates)
13) mathematical astrology.

As noted above, already in the eighth century in India and Afghanistan there were compiled a number of Arabic *zīj*es. These earliest examples, based on Indian and Sasanian works, are lost, as are the earliest examples compiled at Baghdad in the eighth century. With the *zīj*es compiled in Baghdad and Damascus in the early ninth century under the patronage of the Caliph al-Ma'mūn we are on somewhat firmer ground. These follow either the tradition of the *Almagest* and *Handy Tables* or the Indian tradition. Manuscripts exist of the *Mumtaḥan Zīj* of Yaḥyā ibn Abī Manṣūr and the Damascus *Zīj* of Ḥabash, each of which was based on essentially Ptolemaic theory rather than Indian. The *Zīj* of al-Khwārizmī, based mainly on the Persian and Indian traditions, has survived only in a Latin translation of an Andalusian recension. The *Ṣābi' Zīj* of al-Battānī of Raqqa *c.* 910; the *Ḥākimī Zīj* of Ibn Yūnus, compiled in Cairo at the end of the tenth century; the *zīj* called *al-Qānūn al-Mas'ūdī* by al-Bīrūnī, compiled in Ghazna about 1025; the *Zīj* of Ibn Isḥāq, compiled in Tunis *c.* 1195; the *Īlkhānī Zīj* of Naṣīr al-Dīn al-Ṭūsī, prepared in Maragha in the mid thirteenth century; and the *Sulṭānī Zīj* of Ulugh Beg from early fifteenth-century Samarqand: these are amongst the most important later works of this genre, and also the most influential.

The only *zīj*es from the early period of Islamic astronomy that have been published with translation and commentary are those of al-Khwārizmī (in the much modified later recension) and al-Battānī. The Arabic text of the *Zīj* of al-Bīrūnī has been published and a Russian translation and commentary are available. The observation accounts in the introduction of the *Ḥākimī Zīj* of Ibn Yūnus and the texts (but not the tables) of the *Zīj*es of Ibn al-Bannā' (Marrakesh, *c.* 1300) and of Ulugh Beg have been published and translated. Also a Byzantine translation of one of the *zīj*es of al-Fahhād (Iran, *c.* 1150; see p. 108) has been published. No other *zīj*es have received such attention.

Although the *zīj*es are amongst the most important sources for our knowledge of Islamic mathematical *astronomy*, it is important to observe that they generally contain extensive tables and explanatory text relating to mathematical *astrology* as well. Islamic astrological texts form an independent corpus of literature, mainly untouched by modern scholarship. Often highly sophisticated mathematical procedures are involved. It should also be pointed out that in spite of the fact that astrology was anathema to Muslim orthodoxy, it has always been (and still is) widely practised in Islamic society.

We shall now consider various aspects of the *zīj*es and the related literature.

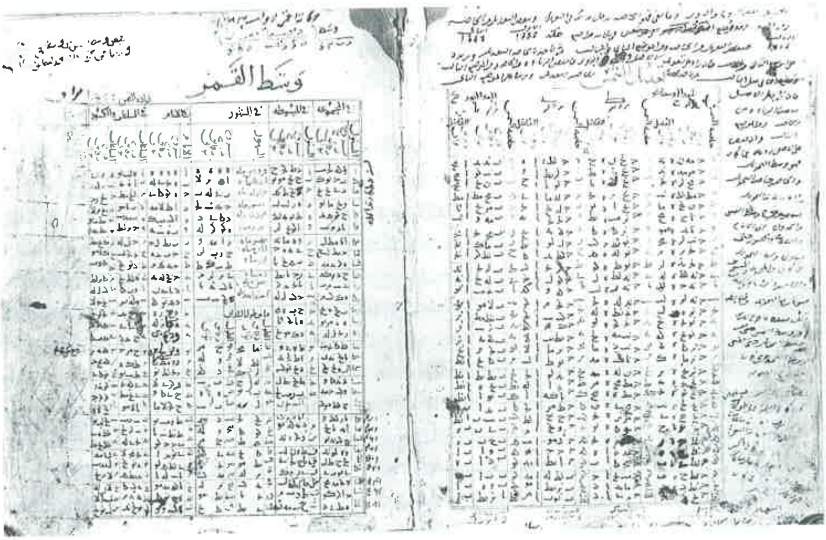
NUMERICAL NOTATION AND BASIC MATHEMATICAL AUXILIARY TABLES All early Islamic astronomical tables have entries written in Arabic alphanumerical notation and expressed sexagesimally, that is, to base 60. A number written in letters equivalent to '23 30 17 seconds' (Ulugh Beg's value for the obliquity) stands for $23 + \frac{30}{60} + \frac{17}{3600}$ degrees, that is, 23°30'17". In sexagesimal arithmetic, more so than in decimal arithmetic, it is useful to have a multiplication table at hand, and such tables, with 3,600 or even 216,000 entries, were available.

Already in the early ninth century Muslim astronomers had restyled the cumbersome Indian sine function using the Greek base 60 (which the Greeks had used for their even more cumbersome chord function). Likewise the Indian shadow functions, unknown in Greek astronomy, were adopted with different bases (12, 6, $6\frac{1}{2}$ and 7, and also 60 and occasionally 1). Most *zīj*es contain tables of the sine and (co)tangent function for each whole, or half, or quarter degree of arc. Entries are generally given to three sexagesimal digits, corresponding roughly to five decimal digits. But certain Muslim scholars compiled more extensive sets of trigonometric tables that were not included in *zīj*es. Already in the early tenth century al-Samarqandī prepared a set of tables of the tangent function with entries to three sexagesimal digits for each minute of arc. Later in the same century Ibn Yūnus tabulated the sine function to five sexagesimal digits, equivalent to about nine decimal digits, for each minute of arc, also giving the differences for each second. He also tabulated the tangent function for each minute of arc, and the solar declination for each minute of solar longitude. His trigonometric tables were not sufficiently accurate to warrant this number of significant figures, and indeed over four centuries were to elapse before the compilation in Samarqand of the magnificent trigonometric tables in the *Sulṭānī Zīj* of Ulugh Beg, which display the values of the sine and tangent to five sexagesimal digits for each minute of argument and are generally accurate in the last digit.

PLANETARY TABLES AND EPHEMERIDES Given the Ptolemaic models and tables of the mean motion and equations of the sun, moon and planets such as were available to Muslim astronomers in the *Almagest* and *Handy Tables*, or the corresponding tables based on Indian models that exemplify the *Sindhind* tradition, Muslim astronomers from the ninth to the sixteenth century sought to improve the numerical parameters on which these tables were based. Most of the leading Muslim astronomers of the early period made solar observations and computed new solar equation tables. Ibn Yūnus is the only astronomer from the first four centuries of Islam known to have compiled a new set of lunar equation tables. The majority of Islamic planetary equation tables are Ptolemaic, and where exceptions do occur, such as in the tables of Ibn al-Aʿlam and Ibn Yūnus for Mercury, we find that they are based on a Sasanian parameter rather than on any new observations. Pending a new edition of Kennedy's survey of Islamic *zīj*es to include all available parameters as well as bio-bibliographical data and a reclassification of the 200-odd known *zīj*es into clearly defined families, it is better not to say more at this time.

Ptolemy used the same data as Hipparchus for his determination of the solar apogee and hence obtained the same result. The Muslims thus inherited the notion that the solar apogee is fixed with respect to the fixed stars (although the planetary apogees move with the motion of precession), and it is to their credit that their earliest observations established that the solar apogee had moved about 15° since the time of Hipparchus. Most early Muslim astronomers accepted the *Mumtaḥan* value of 1° in $66\frac{2}{3}$ Persian years (actually a parameter attested in earlier Persian sources) for both precession and the motion of the apogees. Ibn Yūnus possessed all the necessary data that could be used to demonstrate that the motion of the solar apogee is not the same as the motion due to precession, but he chose to use the same value for both, 1° in $70\frac{1}{4}$ Persian years, which happens to be remarkably close to the actual rate of precession. Al-Bīrūnī (Central Asia, *c.* 1025) seems to have been the first to distinguish the proper motion of the solar apogee from the motion of precession (this discovery is sometimes erroneously attributed to al-Battānī). It was Ibn al-Zarqāllu (Andalusia, *c.* 1070) who was the first to assign a numerical value to both motions, although he also subscribed to the theory of trepidation.

All Islamic *zīj*es contained tables of mean motions and equations for computing solar, lunar and planetary positions for a given time (Fig. 50). Some of the equation tables are arranged in a form more convenient for the user (so that one simply has to enter the mean motions, and calculations are avoided). Auxiliary tables were sometimes available for generating ephemerides without the tedious computation of daily positions from mean-motion and equation tables. From the ninth to the nineteenth century Muslim astronomers compiled ephemerides displaying solar, lunar and planetary positions of each day of the year, as well as information on the new moons and astrological predictions resulting from the position of the moon relative to the planets. Al-Bīrūnī described in detail how to compile ephemerides in his astronomical and astrological



50 Astronomical tables typical of the type found in Islamic *zījes*. These serve the solar equation (right) and the lunar mean motion (left). They are found in a Yemeni manuscript of the *Zīj* of the Persian astronomer Kūshyār ibn Labbān, compiled *c.* AD 1000 and copied here *c.* AD 1250. The manuscript has various marginalia giving modifications to the tables for the longitude of Sanaa. The Yemen was an important centre of astronomy in the Middle Ages. (Egyptian National Library, Cairo, MS DM 400)

handbook *Instruction in the Art of Astrology* (*Tafhīm*). Manuscripts of ephemerides had a high rate of attrition since the tables could be dispensed with at the end of the year: the earliest complete extant examples are from fourteenth-century Yemen, discovered in Cairo in the 1970s and still unpublished; on the other hand, literally hundreds of ephemerides survive from the late Ottoman period.

STELLAR CO-ORDINATES AND URANOGRAPHY Most *zījes* contain lists of stellar co-ordinates in either the ecliptic or the equatorial system, or occasionally in both systems. A survey of the stellar co-ordinates in Islamic *zījes*, which has not yet been conducted, would be a valuable contribution to the history of Islamic astronomy, and could help determine the extent to which original observations were made by Muslim astronomers. An impressive amount of research on Arabic star names and their later influence in Europe has been conducted in the last few years by P. Kunitzsch.

In his *Book of Constellation Figures* (*Ṣuwar al-kawākib*, Col. Pl. IX) the tenth-century Shiraz astronomer al-Ṣūfī presented lists of stellar co-ordinates as well as illustrations of the constellation figures from the Hellenistic tradition and also information on the lunar mansions following the Arab tradition. Later Islamic works on uranography are mostly restricted to Persian and Turkish translations of al-Ṣūfī, although some astrological works also contain illustrations of the constellations that have recently attracted the attention of historians of Islamic art.

SPHERICAL ASTRONOMY AND SPHERICAL TRIGONOMETRY Most *zīj*es contain in their introductory text the solutions of the standard problems of spherical astronomy, such as, to give only one example, the determination of time from solar and stellar altitude. Rarely is any explanation given of how the formulae outlined in words in the text were derived. There were two main traditions. In the first, the problems relating to the celestial sphere are reduced to geometric or trigonometric problems on a plane. The construction known as the analemma was a singularly powerful tool for solutions of this kind. In the second, the problems are solved by applications of rules of spherical trigonometry. Both techniques are ultimately of Greek origin, and Muslim scholars made substantial contributions to each.

There is some confusion about these contributions in the modern literature. It has been assumed by modern writers that when a medieval writer used a medieval formula that is mathematically equivalent to the modern formula derived by a specific rule of spherical trigonometry, the medieval scholar must have known the equivalent of the modern rule of spherical trigonometry. In fact, however, the medieval formula may have been derived without using spherical trigonometry at all. The first known Islamic treatise dealing with spherical trigonometry independently from astronomy is by the eleventh-century Andalusian Ibn Muʿādh. The contributions to spherical astronomy by such scholars as Thābit ibn Qurra, al-Nayrīzī, Abu l-Wafāʾ al-Būzajānī, al-Khujandī, Kūshyār ibn Labbān, al-Sijzī, Abū Naṣr, are outlined in the recently rediscovered *Keys to Astronomy* (*Maqālīd*) of al-Bīrūnī, also from the eleventh century.

Already in the work of Ḥabash in the mid ninth century we find a Muslim astronomer at ease with both spherical trigonometric methods and analemma constructions for solving problems of spherical astronomy. In the *zīj*es of scholars of the calibre of Ibn Yūnus and al-Bīrūnī we find various methods for solving all of the standard problems of medieval spherical astronomy. The auxiliary trigonometric tables compiled by such scholars as Ḥabash, Abū Naṣr (*c.* 1000) and al-Khalīlī (*c.* 1360) for solving all of the problems of spherical astronomy for any latitude are a remarkable testimony to their mastery of the subject.

Applications of astronomy to aspects of religious practice

The lunar calendar

The Muslim calendar is lunar and the civil months begin with the first sighting of the lunar crescent. The precise determination of the beginnings and ends of the months is particularly important for Ramaḍān, the sacred month of fasting, and various other religious festivals. The legal scholars were content to rely on direct sighting of the crescent or on alternating 29 and 30-day months to regulate the calendar. But the subject of lunar crescent visibility was generally treated in *zījes* (Fig. 51), and a wide variety of methods and tables were devised to facilitate the solution of this problem. Al-Khwārizmī, for example, compiled a table of the minimum ecliptic elongation of the sun and moon for each zodiacal sign, computed for the latitude of Baghdad, and another early Muslim astronomer, perhaps from Andalusia, compiled a similar table for each of the climates, no doubt inspired by Ptolemy's planetary visibility tables. A few early Islamic astronomers, notably Thābit ibn Qurra and Ibn Yūnus, postulated conditions that appear to be considerably more sophisticated, although they are not yet fully understood. Others like al-Battānī and Ibn al-Zarqāllu, merely played around with numbers, deriving complicated procedures from a basic notion that visibility occurs when the ecliptic elongation is 12° or 1 day's relative motion of the sun and moon after conjunction. Unfortunately no Muslim astronomer has left us any observational data on crescent visibility, and with a few possible exceptions yet to be identified none of the various sets of conditions proposed by the Muslim astronomers appears to be based on such data.

Nowadays there is often confusion about the beginning of Ramaḍān, such as could never have occurred in medieval times. This confusion results from the fact that the crescent may be seen in some locations and not in others, and no less from the reluctance of the religious scholars, who have the final say in announcing the new month, to listen to the astronomers.

51 Extracts from an Egyptian set of tables displaying calculations for the visibility of the lunar crescent. These display for each month of the lunar years 1125 and 1126 Hijra (= AD 1713–14), for sunset on the first day of each month in the civil calendar, the lunar longitude and latitude, angular separation of the sun and moon, and difference in their setting times, followed by a prediction: '[the crescent] will be seen clearly', 'probably' (literally, mostly), 'with difficulty', 'not at all'. In the last case, the month would officially begin the next evening. (Egyptian National Library, Cairo, MS DṢ 155)

The times of prayer

In Islam the times of prayer are astronomically determined. The standard definitions of the five prayer-times, still in use today, are briefly as follows: The Muslim day begins at sunset, and the interval during which the first prayer (*maghrib*) is to be performed lasts from sunset to nightfall. The interval for the second prayer (*ʿishāʾ*) begins at nightfall and lasts until daybreak. The third prayer (*fajr*) is performed in the interval between daybreak and sunrise. The permitted time for the fourth prayer (*ẓuhr*) begins when the sun has crossed the meridian and ends when the interval for the fifth prayer (*ʿaṣr*) begins, namely, when the shadow of an object equals its midday shadow increased by the length of the object. The interval for the fifth prayer may last until the shadow increases again by the length of the object or until sunset. In medieval Andalusian practice the fourth prayer begins when the shadow has increased beyond its midday minimum by one quarter of the length of the object. Isolated instances of the performance of another prayer (*ḍuḥā*) in the morning, at a time after sunrise equal to the time remaining till sunset after the *ʿaṣr*, are attested in medieval sources. The definitions of the daytime prayers in terms of shadow increases relate the times to the seasonal hours (one-twelfth divisions of the length of daylight), the connection being provided by an approximate Indian formula for time-keeping known to the Muslims in the eighth century. This formula associates the end of the ninth seasonal hour (mid-afternoon) with a shadow increase of 1 gnomon-length, and the end of the tenth hour with a shadow increase of 2 gnomon-lengths.

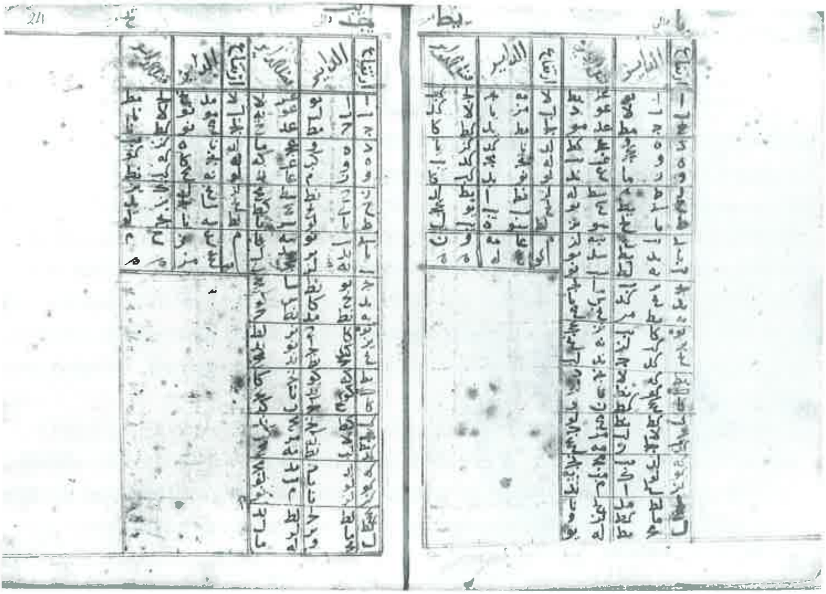
Clearly the times of the prayers as defined above can be regulated without difficulty by observation, assuming a clear sky. A genre of literature dealing with time-keeping from a non-mathematical point of view was compiled by specialists in folk astronomy, some of whom were also noted legal scholars. These works consist mainly of discussions of Quranic verses and Prophetic statements on these two subjects, embellished with descriptions of simple procedures for time-keeping by shadows and by the lunar mansions. Most surviving examples are of thirteenth-century or later Yemeni and Hejazi provenance, but the trend was set already in the ninth century. Since they involved the religious traditions, they formed part of the corpus of Islamic legal literature, and they were subject to different interpretations by different legal schools. In the first few centuries of Islam, as far as we can tell from the scant evidence available, the times of prayer were regulated by the muezzins themselves, and these were selected as much because of their fine voices as for their ability in folk astronomy.

This notwithstanding, it was found convenient to have tables at hand displaying the length of the prayer-times for each day of the year or each degree of solar longitude. The earliest such tables we have are a set for Baghdad displaying the shadow at the *ẓuhr* and the *ʿaṣr* for each 6° of solar longitude, and another, found only in a thirteenth-century Iraqi *zīj*, but probably also dating from the ninth century, displaying the time from sunset to daybreak (based on an approximate Indian formula for time-keeping) and the midday shadow, and the solar altitudes at midday and the beginning and end of the *ʿaṣr* for each day of the Syrian calendar. About the middle of the tenth century ʿAlī ibn Amājūr compiled two tables displaying the time of day as a function of solar meridian altitude and instantaneous altitude. The first, based on an accurate formula, was computed specifically for Baghdad, and the second, based on an approximate Indian formula, is universal and works quite well for all latitudes. In both cases values are given in seasonal day-hours and minutes for each degree of both arguments.

These early Islamic tables for time-keeping, of which so few examples survive, began a tradition that reached its zenith in thirteenth-century Cairo and fourteenth-century Damascus (Fig. 52). Most of the corpora of tables for time-keeping compiled for these two centres and others such as Jerusalem (Fig. 53), Alexandria, Maragha, Tunis and Taiz, belong to the later period of Islamic astronomy. Some remarkable tables were produced: one, compiled in Egypt in the late thirteenth century, has three arguments (solar or stellar altitude and meridian altitude and half arc of visibility) and displays the time of day or night for any terrestrial latitude; it has over 400,000 entries. It has only recently become apparent that European astronomers in later centuries compiled tables similar in their conception; few of these have been studied yet. Indeed the history of such tables extends from the ninth to the nineteenth century in the Islamic world and from the fourteenth century onwards in Europe and eventually also North America.

It was apparently in Egypt in the thirteenth century that the office of the *muwaqqit* or mosque astronomer responsible mainly for the times of prayer was developed. Most

52 An extract from the prayer-tables for the latitude of Damascus, computed in the mid fourteenth century by the *muwaqqit* al-Khalīlī. Twelve functions relating to time-keeping are tabulated across the page for each degree of solar longitude (corresponding roughly to each day of the year); this double page serves the sign of Aquarius. Similar tables have been found for localities between Morocco and Central Asia, Crete and the Yemen. Such tables enabled the *muwaqqits* to inform the muezzins when the time for each of the five prayers had arrived so that they could summon the faithful to prayer. (Bibliothèque Nationale, Paris, MS ar. 2558)



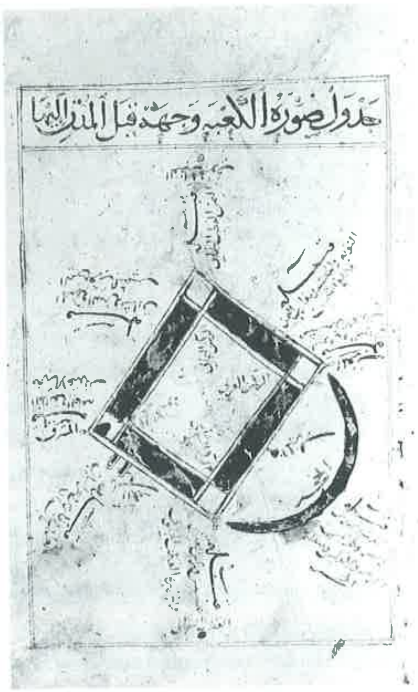
53 An extract from the tables for time-keeping for the sun, compiled for the latitude of Jerusalem by the fourteenth-century *muwaqqit* al-Karakī. For each degree of solar longitude (here Aquarius 11° and 12°, also serving Scorpio 19° and 18°) and for each degree of solar altitude up to the maximum, the time since sunrise and hour-angle are tabulated in equatorial degrees and minutes. The first tables of this kind were prepared in Baghdad in the ninth century and numerous examples for other locations were compiled during the next millennium. (Leipzig University Library, MS 808)

of the Egyptian and Syrian astronomers of consequence from the fourteenth and fifteenth centuries were *muwaqqits*. The office was particularly important in the Ottoman Empire and in many mosques from Anatolia to the Balkans one can still see the buildings attached to mosques where the *muwaqqits* kept their books and instruments. There does not appear to have been a similar office in the Muslim East, but we do have evidence that there was a team of *muwaqqits* in Granada around 1300.

The tables that modern Muslims use to regulate their prayers, published in newspapers, pocket diaries and wall calendars, have a history of over a millennium, only recently documented. The muezzin's call to prayer at five specific times is one of the most distinctive features of modern Islamic life.

The sacred direction

The Quranic injunction that prayer and other ritual acts should be performed facing the Kaaba in Mecca caused no problems for the first generations of Muslims who found



54 An illustration of the Kaaba, with each of the corners and four walls associated with regions of the Islamic commonwealth, and the *qibla* in each region defined in terms of an astronomical horizon phenomenon. From a copy of the cosmography of the fifteenth-century Syrian Ibn-al-Wardī. (Bibliothèque Nationale, Paris, MS ar. 2186)

themselves in localities as far apart as Andalusia and Sind. They had not the means to determine the *qibla*, or direction of Mecca, in the localities they settled, so they simply used the direction of the pilgrim road to Mecca or the cardinal directions; as a result, the earliest mosques and the *miḥrāb*s in their *qibla* walls face Mecca only roughly. But the Muslims knew that the Kaaba was aligned in certain astronomical directions (major axis towards the rising of Canopus, minor axis solstitially aligned), and for them the Kaaba was at the centre of their world (Fig. 54). They devised a simple expedient to face the segment of the perimeter corresponding to their location: one should stand in the same direction as one would be standing in if one were actually in front of that segment of the perimeter of the Kaaba. Different schemes of sacred geography were developed by the specialists in folk astronomy and the legal scholars, in which the world was divided into sectors about the Kaaba and the *qibla* in each sector was defined in terms of the rising or setting of the sun or a certain fixed star. This kind of information about the *qibla* was proposed by the legal scholars, and is reflected in the orientations of medieval mosques, only a minority of which are aligned in directions that could have been proposed by the astronomers.

By the early ninth century Ptolemy's list of geographical co-ordinates was available to Muslim scholars and the co-ordinates of Mecca and Baghdad had been investigated by a team commissioned by the Caliph al-Ma'mūn. Further, exact geometrical and trigonometric procedures had been devised for determining the *qibla* from geographical co-ordinates. (The problem of the determination of the *qibla* is easily transformed into a problem of spherical astronomy by considering the zeniths of the localities involved.) al-Bīrūnī's treatise on mathematical geography (*Taḥdīd*), the ultimate goal of which was the determination of the *qibla* at Ghazna, is the most important work of its kind from the medieval period. Since these exact procedures for determining the *qibla* were rather complicated, approximate methods were also derived. Already in the ninth century a table was compiled based on one such method and it displays the direction of Mecca as a function of terrestrial latitude and longitude. Over the centuries several such tables were compiled. We return later to some of the various instruments used for finding the *qibla*.

Only in the eighteenth century did it become possible for the first time to measure longitude differences correctly, and only then could it become obvious that most medieval longitude co-ordinates were incorrect and that even the *qibla*s derived by a correct mathematical procedure but based on these coordinates were also off by a few degrees. Some medieval mosques that are still in use have two *miḥrāb*s indicating the *qibla*, the second being based on modern calculations. Nowadays some Muslims use pocket-compasses provided with lists of *qibla*s for the major cities of the world.

Observation programmes and regional schools of astronomy

Al-Ma'mūn's circle

In the early ninth century the Abbasid Caliph al-Ma'mūn patronised observations first in Baghdad and then in Damascus, gathering the best available astronomers to conduct observations of the sun and moon. Some of the results were incorporated into a *zīj* called *al-Mumtaḥan*, 'tested', although the details of the activities at the two observation posts are somewhat confusing. The *Mumtaḥan Zīj* was apparently compiled in Baghdad by Yaḥyā ibn Abī Manṣūr, but upon his death, according to Ḥabash, the Caliph ordered his colleague Khālid al-Marwarrūdhī to prepare some new instruments and conduct a one-year programme of solar and lunar observations in Damascus in order to compile a new *zīj*. According to Ḥabash this was done, but no such *zīj* is otherwise known to have been prepared before Ḥabash's own *Damascus Zīj*.

These observations, like later ones, were mainly directed towards determining the local latitude and current value of the obliquity, and towards deriving improved parameters for the Ptolemaic planetary models and more accurate star positions. The armillary sphere, the meridian quadrant and the parallactic ruler were known to the Muslims from the *Almagest*, and they added new scales and other modifications, often building larger instruments even when smaller ones would have sufficed. Our knowledge of the instruments used by al-Ma'mūn's astronomers is meagre. An armillary sphere used by Yaḥyā in Baghdad was said to display markings for each 10' of arc, but even contemporary astronomers were not impressed by the precision of the results obtained using it. A mural quadrant made of marble with a radius of about 5 m was used in Damascus, as well as a vertical gnomon made of iron standing about 5 m high. Al-Ma'mūn also patronised measurements of the longitude difference between Baghdad and Mecca (by simultaneous observations of a lunar eclipse) in order to establish the *qibla* at Baghdad properly, as well as measurements of the length of one degree of terrestrial latitude. Most of what is known about the Baghdad and Damascus observatories is provided by Ibn Yūnus and al-Bīrūnī (see p. 163), and the available manuscripts of the *Zīj*es of Yaḥyā and Ḥabash await detailed study.

Other observational programmes

Besides the officially-sponsored observations conducted in Baghdad and Damascus in the early ninth century, there are numerous instances of other series of observations conducted in different parts of the Muslim world.

The two brothers called Banū Mūsā made observations in their own house in Baghdad and also in nearby Samarra about 30 years after the *Mumtaḥan* observations. They also arranged for simultaneous observations of a lunar eclipse in Samarra and Nishapur in order to determine the difference in longitude between the two cities. In view of their proficiency in mathematics, it is most unfortunate that neither of the two *zīj*es compiled by them has survived.

Al-Battānī carried out observations during the period 887 to 918 in Raqqa in North Syria. He appears to have financed his observational activity himself, and although we have no description of the site where he made his observations, the instruments mentioned in the *zīj* based on his observations include an armillary sphere and mural quadrant, as well as a parallactic ruler, an astrolabe, a gnomon and a horizontal sundial.

The observational activities of the Baghdad family known as the Banū Amājūr were almost contemporary with those of al-Battānī in Raqqa. Father and two sons, and also a freed family slave, all made observations and each compiled a *zīj*, none of which survives. In the accounts of their eclipse observations recorded by Ibn Yūnus it appears that the place where they conducted their observations had some kind of a balcony fitted with slits for observation, but the details are obscure. A particularly interesting account of a solar eclipse in the year 928 that they observed by reflection in water includes a remark that the altitude of the sun was measured on an instrument marked for each third of a degree.

A large mural quadrant was erected at Rayy (near modern Tehran) about the year 950 but we have information only on its use to establish the local latitude and obliquity of the ecliptic. In Shiraz not long thereafter al-Ṣūfī used an armillary sphere with diameter about 5 m to derive the same parameters and to 'observe' equinoxes and solstices. Al-Ṣūfī is best known for his work on the fixed stars, but it seems that this was based more on ‘observation’ with the naked eye than on ‘measurement’, looking at the heavens with precision instruments and making estimates of positions. Another contemporary astronomer who conducted observations on which we have no information other than the main parameters of his *zīj* was Ibn al-Aʿlam. The observations of both al-Ṣūfī and Ibn alʿAlam were patronised by the Buwayhid ruler ʿAḍud al-Dawla.

In the late tenth century the distinguished mathematician and astronomer Abu l-Wafā’ al-Būzajānī made observations in Baghdad. Most of these appear to have been directed towards the determination of the solar parameters, and the obliquity of the ecliptic and the latitude of Baghdad, although Abu l-Wafā’ also collaborated with al-Bīrūnī in Khwarizm (modern Khiva in Uzbekistan) on the simultaneous observation of a lunar eclipse in the year 997. We have no information on the nature of the site where Abu l-Wafā’ made his observations, other than its location in a specific quarter of Baghdad.

Contemporaneous with the activity of Abu l-Wafā’ was the establishment in 998 of an observatory in the garden of the Baghdad residence of the Buwayhid ruler Sharaf al-Dawla. The organisation of a building and programme of observations was entrusted to Abū Sahl al-Qūhī, a mathematician of considerable standing. We know from contemporary historical records that a special building was erected for the observations, which in turn were witnessed by ‘judges, scientists and scholars of note, astronomers, and engineers’. In view of the favourable conditions under which this observatory was established, and the competence of its director, it is somewhat surprising that the two recorded ‘observations’ that were ‘witnessed’ by so many dignitaries were the entry of the sun into Cancer and Libra in the year 988. Al-Bīrūnī describes the main instrument which was constructed as a hemisphere of radius 12.5 m on which the solar image was projected through an aperture at the centre of the hemisphere. Activity at the observatory stopped in 989 with the death of Sharaf al-Dawla, so that the institution lasted not much more than a year.

In 994 Abū Maḥmūd al-Khujandī made a measurement of the obliquity using a meridian sextant of about 20 m radius. This instrument was erected in Rayy but al-Khujandī confessed to al-Bīrūnī that it was so large that the centre of the sextant had become displaced from its intended position.

The Egyptian astronomer Ibn Yūnus made a series of observations of eclipses, conjunctions and occultations, as well as equinoctial and solstitial observations. We are extremely fortunate to have not only his reports of these observations but also his citations of earlier observations of the same kind made by individuals such as Ḥabash and the Banū Amājūr. Ibn Yūnus’s purpose in making these observations and recording them in the introduction to his *Zīj* is somewhat obscured by the fact that he does not list those observations or present those calculations with which he derived his new solar, lunar and planetary parameters. Neither does he mention any locations for his observations other than his grandfather’s house in Fustat and a nearby mosque in al-

Qarāfa. The popular association of Ibn Yūnus with an observatory on the Muqaṭṭam Hills outside Cairo is, as A. Sayılı has shown, a myth. Nevertheless, Ibn Yūnus mentions at least one instrument, probably a meridian ring, that was provided by the Fatimid Caliphs al-ʿAzīz and al-Ḥākim. In a later medieval Egyptian source Ibn Yūnus is reported to have received 100 dinars a day from al-Ḥākim, and it may be that such extremely high payments were made to Ibn Yūnus when he was making satisfactory astrological predictions for the Caliph. Al-Ḥākim made an abortive attempt to found an observatory in Cairo, but this was after the death of Ibn Yūnus in 1009. At some time during his reign there was an armillary sphere in Cairo with nine rings, each large enough that a man could ride through them on horseback.

The observations of al-Bīrūnī were conducted between 990 and *c.* 1025 in several localities between Khwarizm (modern Khiva) and Kabul. His recorded observations include determinations of equinoxes and solstices, eclipses, and determinations of the obliquity and local latitude.

The corpus of tables known as the *Toledan Tables* was compiled in the eleventh century, based on observations directed by Ṣāʿid al-Andalusī and continued by Ibn al-Zarqāllu. Only the mean motion tables in this corpus of tables are original; most of the remainder were lifted from the *Zīj*es of al-Khwārizmī and al-Battānī.

In the thirteenth century there was a substantial observational programme at Maragha. The results are impressive only in so far as theoretical astronomy is concerned (see p. 150). Otherwise the trigonometric and planetary tables in the major production of the Maragha astronomers were modified or lifted *in toto* from earlier sources. This is not a happy outcome for a generously endowed observatory fitted with the latest observational instruments, known to us only from texts. In the early fifteenth century the scene had moved to Samarqand in Central Asia: there a group of astronomers directed by the astronomer-prince Ulugh Beg did impressive work. Only the 40-m meridian sextant survives from the observatory. These men produced a set of tables which, however, it would be foolish to judge before they have been properly studied. The same is true for the short-lived observatory in Istanbul under the direction of Taqi l-Dīn (1577; see frontispiece).

Regional schools of astronomy

After the tenth century there developed regional schools of astronomy in the Islamic world, with different interests and concentrations. They also had different authorities (for example, in the furthest East al-Bīrūnī and al-Ṭūsī, and in Egypt Ibn Yūnus). The main regions were Iraq; Iran and Central Asia; Muslim Spain; Egypt and Syria; the Yemen; the Maghrib; and later also the Ottoman lands. Only recently have the complex tradition of Muslim Spain (tenth–fourteenth centuries), the colourful tradition of Mamluk Egypt and Syria (thirteenth–early sixteenth centuries), the distinctive tradition of Rasulid Yemen (thirteenth–sixteenth centuries), and the staid tradition of the Maghrib (twelfth–nineteenth centuries) been studied. The traditions of Ottoman Turkey and Mogul India are currently being researched.

Astronomical instruments

As noted in the previous section most Islamic observational instruments are lost and known to us only through texts. The state of documentation of the other, smaller Islamic astronomical instruments that do survive leaves much to be desired. Many of the most important instruments are still unpublished, and much that has been written on instruments is on a very amateur level. For these reasons a project is currently underway in Frankfurt to catalogue all Islamic instruments (and European ones) to *c.* 1550 as well as various historically significant later Islamic pieces.

Also the most important writings on instruments have not yet received the attention they deserve. For example, a hemispherical observational instrument for a fixed latitude was devised by the tenth-century astronomer al-Khujandī, the leading instrument-maker of the early period, and this was modified in the twelfth century to serve all latitudes. There are no surviving examples and the available manuscripts have yet to be studied. An important work on instruments was compiled in Cairo *c.* 1280 by Abū ʿAlī al-Marrākushī; this has yet to be subjected to a detailed analysis. The author simply collected all of the treatises on instruments known to him and incorporated them into his book. An exciting find of the 1980s was a treatise by the fourteenth-century Aleppo astronomer Ibn al-Sarrāj, the leading instrument-maker of the later Islamic period. In this the author described every kind of instrument known to him as well as those invented by himself. This treatise is currently being studied.

Armillary spheres and globes

In the eighth century al-Fazārī wrote a treatise on the armillary sphere, called in Arabic *dhāt al-ḥalaq*, which means ‘the instrument with the rings’. No early Islamic armillary spheres survive, but several other treatises on it were compiled over the centuries. The earliest treatise in Arabic dealing with the celestial globe, called *dhāt al-kursī*, ‘the instrument with the stand [literally, the ‘throne’ on which it sits]’ or simply *al-kura*, ‘sphere’, was written by Qusṭā ibn Lūqā in the ninth century. This treatise by Qusṭā, who was one of the most important translators of Greek works into Arabic, remained popular for a millennium. Of the various surviving celestial globes, which number over 100, none predates the eleventh century. A globe made in Mosul in 1274–5 is preserved in the British Museum (Col. Pl. X).

The spherical astrolabe, unlike the armillary sphere and the celestial globe, appears to be an Islamic development. Various treatises on it were written from the tenth to the sixteenth century, and only one complete instrument, from the fourteenth century, survives. Ḥabash wrote on the spherical astrolabe, the armillary sphere, and the celestial globe, as well as on various kinds of planispheric astrolabes.

Astrolabes

Al-Fazārī also wrote on the use of the astrolabe. The tenth-century bibliographer Ibn al-Nadīm states that al-Fazārī was the first Muslim to make such an instrument; he also informs us that at that time the construction of astrolabes was centred on Harran and spread from there. Several early astronomers, including Ḥabash, al-Khwārizmī and al-Farghānī, wrote on the astrolabe, and introduced features not found on earlier Greek instruments, such as the shadow squares and trigonometric grids on the backs and the azimuth curves on the plates for different latitudes, as well as the universal plate of horizons. Also extensive tables were compiled in the ninth century to facilitate the construction of astrolabes.

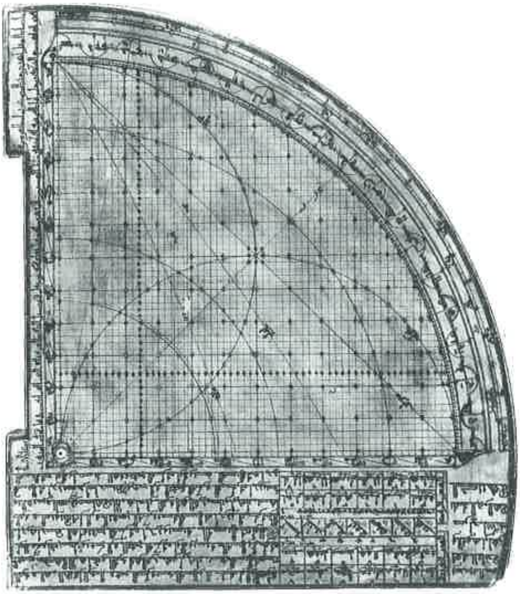
Another important development to the astrolabe occurred in Andalusia in the eleventh century, when Ibn al-Zarqāllu devised the single universal plate (*ṣafīḥa*) called *shakkāziyya* and the related plate called *zarqālliyya* with two sets of *shakkāziyya* markings for both equatorial and ecliptic co-ordinate systems. The latter was fitted with an alidade bearing a movable perpendicular straight-edge (transversal). Several treatises on these two instruments exist in both Western and Eastern traditions of later Islamic astronomy; the Europeans knew of them as the *saphea*. Ibn al-Zarqāllu's contemporary, 'Alī ibn Khalaf, wrote a treatise on a universal astrolabe that did not need plates for different latitudes. This treatise exists only in Old Spanish in the *Libros del Saber*, and was apparently

55 The front of the universal astrolabe of Ibn al-Sarrāj, dated AD 1329. This remarkable instrument not only represents the culmination of Islamic astrolabe-making, but has no equal in sophistication amongst instruments from the European Renaissance. Whereas the standard astrolabe requires a different plate for each latitude, that of Ibn al-Sarrāj has plates that serve all latitudes; indeed, the various components can be used in five different ways to solve all the problems of spherical astronomy for any latitude. IC no. 140 (Benaki Museum, Athens) not known in the Islamic world outside Andalusia. The instrument was further developed in Syria in the early fourteenth century: Ibn al-Sarrāj devised in Aleppo a remarkable astrolabe that can be used universally in five different ways (Fig. 55).

The astrolabes made by Muslim craftsmen show a remarkable variety within each of several clearly defined regional schools. We may mention the simple, functional astrolabes of the early Baghdad school; the splendid astrolabe of al-Khujandī of the late tenth century, which started a tradition of zoomorphic ornamentation that continued in the Islamic East and in Europe for several centuries; the very different astrolabes of the Andalusian school in the eleventh century and the progressive schools of Iran in the thirteenth and fourteenth centuries; and the remarkable instruments from Mamluk (thirteenth and fourteenth-century) Egypt and Syria. The British Museum possesses a spectacular Mamluk astrolabe by ʿAbd al-Karīm al-Miṣrī. After about 1500 the construction of astrolabes continued in the Maghrib, in Iran and in India until the end of the nineteenth century. Many of these, especially those from the Islamic East, were objects of the finest workmanship. Such is the splendid astrolabe made in Isfahan in 1712 for Shāh Ḥusayn, also now in the British Museum (Col. Pl. XI).

Quadrants

Another category of observational and computational devices to which Muslim astronomers made notable contributions was the quadrant, of which we can distinguish three main varieties. First, the sine quadrant with an orthogonal grid. This instrument, in a simpler form, was described already by al-Khwārizmī and was widely used throughout the Islamic period. Some Islamic astrolabes display such a trigonometric grid on the back. The grid can be used together with a thread and movable marker (or the alidade of an astrolabe) to solve all of the standard problems of spherical astronomy for any latitude. Second, the horary quadrant with fixed or movable cursor. This instrument is described already in an anonymous ninth-century Iraqi source and was likewise commonly used for centuries (albeit usually without the cursor, which is not essential to the function of the device). A set of arcs of circles inscribed on the quadrant display graphically the solar altitude at the seasonal hours (approximately, according to an Indian formula). Other Islamic quadrants from the ninth century onwards had markings for the equinoctial hours. The instrument can be aligned towards the sun so that the time can be determined from the observed altitude using the grid. Again, this kind of markings was often marked on the back of astrolabes. Third, the astrolabic quadrant displaying one half of the altitude and azimuth circles on an astrolabe plate for a fixed latitude, and a fixed ecliptic. The effect of the daily rotation is achieved by a thread and bead attached at the centre of the instrument rather than by the movable astrolabe rete. The quadrant with astrolabic markings on one side and a trigonometric grid (Fig. 56) on the other generally replaced the astrolabe all over the Islamic world (with the notable exceptions of Iran, India and the Yemen) in the later period of Islamic astronomy.

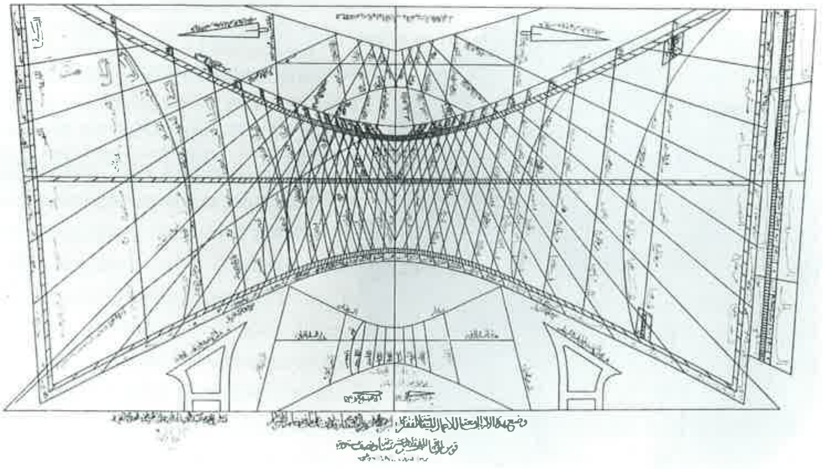


56 A trigonometric grid for solving all the various problems of spherical astronomy without calculation. The grid, a quarter-circle of sexagesimal orthogonal markings like modern graph-paper and with radius 60 units, was developed from simpler varieties first used in Baghdad in the early ninth century. One medieval European quadrant with similar markings has been preserved. This grid, found on an astrolabic quadrant made in Damascus *c.* AD 1800, is remarkable for all the additional markings, lines, arcs of circles and curves, which facilitate the solution of specific problems relating to the astronomically defined times of prayer. (Private collection)

Sundials

We learn from Islamic tradition that the pious Umayyad Caliph ʿUmar ibn ʿAbd al-ʿAzīz (Damascus, *c.* 718) used a sundial, probably a Graeco-Roman one, to regulate the times of the daytime prayers in terms of the seasonal hours. The earliest sundials described in the Arabic astronomical sources are planar, usually horizontal, but also vertical and polar. The mathematical theory for computing the shadow for the seasonal hours at different times of the year and the corresponding azimuths was available from Indian sources, which seem to have inspired the Islamic tradition more than any of the available Greek works. Already the treatise on sundial construction by al-Khwārizmī contains extensive tables displaying the polar co-ordinates of the intersections of the hour lines with the solstitial shadow traces on horizontal sundials for twelve different latitudes. The treatise on sundial theory by Thābit ibn Qurra contains all the necessary mathematical theory for constructing sundials in any plane; likewise impressive from a theoretical point of view is the treatise on gnomonics by his grandson Ibrāhīm.

The earliest surviving Islamic sundial, apparently made in Córdoba about the year 1000 by the Andalusian astronomer Ibn al-Ṣaffār, displays the shadow traces of the equinoxes and solstices, and the lines for the seasonal hours as well as for the times of the two daytime prayers. There is a world of difference between this simple, carelessly constructed piece and the magnificent sundial made in the late fourteenth century by Ibn al-Shāṭir, so devised that it can be used to measure time with respect to any of the



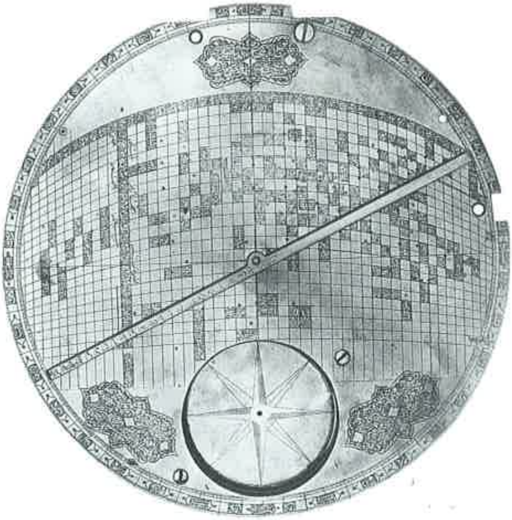
57 The sundial made by Ibn al-Shāṭir for the main minaret of the Umayyad Mosque in Damascus in the year AD 1371–2 was broken by accident in the nineteenth century. The *muwaqqit* who broke it whilst trying to realign it was sufficiently well versed in gnomonics to make this copy, which is still *in situ* on the minaret. Several pieces of the original were discovered in excavations of the drainage system of the mosque in 1958 and are now displayed in the garden of the Archaeological Museum, Damascus. (Alain Brieux, Paris)

five daily prayers (Fig. 57). In the late period of Islamic astronomy a sundial was to be found in most of the major mosques.

Miscellaneous

Several multipurpose instruments were devised by Muslim astronomers. Notable examples are the rule (*mīzān*) of al-Fazārī, fitted with a variety of non-uniform scales for various astronomical functions, and the compendium of Ibn al-Shāṭir, comprising a magnetic compass and *qibla*-indicator, a universal polar sundial, and an equatorial sundial. Of particular interest is a circular brass instrument made in Isfahan *c.* 1700 which is engraved with a world map with Mecca at the centre, and a cartographical grid so devised that the *qibla* for some 150 cities between Spain and China can be read off the outer scale and the distance from Mecca can be read off the non-uniform scale on the diametrical rule (Fig. 58). Muslim interest in projections preserving direction and distance can be traced back several centuries to al-Bīrūnī and Ḥabash.

There are several Islamic treatises on eclipse computers and planetary equatoria (see p. 184) for determining the positions of the planets for a given date. With these the standard problems of planetary astronomy dealt with in *zīj*es are resolved mechanically, without calculation. Treatises on eclipse computers are known from the early tenth



58 A cartographical grid with Mecca at the centre, so devised that one can read the direction and distance to Mecca for any locality in the Islamic commonwealth. The instrument, made in Isfahan *c.* AD 1700 but inspired by an earlier tradition, was originally fitted with a magnetic compass and an hour dial adjustable for any latitude for finding the time of day, all now missing. (Private collection)

century, and al-Bīrūnī in the early eleventh describes such an instrument in detail. A newly discovered manuscript (not yet available for research) contains a treatise by the tenth-century Iranian astronomer Abū Jaʿfar al-Khāzin called *Zīj al-Ṣafāʾiḥ*, the *Zīj of Plates*, describing an equatorium. The sole known example of this instrument, made in the twelfth century, is, alas, incomplete: it is in the form of an astrolabe with tables engraved on the mater and additional markings for the foundation of an equatorium. Otherwise the only known early Islamic treatises on planetary equatoria are from eleventh-century Andalusia. The most interesting aspect of the equatorium described by Ibn al-Zarqāllu is the ellipse drawn on the plate for the centre of the deferent of Mercury; it seems that he was the first to notice this characteristic of Mercury's deferent. Al-Kāshī, the leading astronomer of early fifteenth-century Samarqand, has left us a description of a planetary equatorium with which not only ecliptic longitudes but also latitudes could be determined and eclipses calculated.

Transmission to Europe

The Europeans learned of Islamic astronomy through Spain, a region where, because of political problems and the difficulty of communications, the most up-to-date writings were not always available. This explains, for example, how it came to pass that the Europeans came across two major works of Muslim astronomers from the East, al-Khwārizmī and al-Battānī, at a time when these works were no longer widely used in the Islamic East. It also explains why so few Eastern Islamic works became known in Europe. None of the Eastern Islamic developments to Ptolemy's planetary theory were known in Andalusia or in medieval Europe. Al-Biṭrūjī's unhappy attempt to develop planetary models confused Europeans for several centuries; he must be worth reading, they naïvely thought, because he was trying to reconcile Ptolemy with Aristotle. As far as astronomical time-keeping was concerned, this does not seem to have been of much concern to the Muslims in Spain; hence nothing of consequence was transmitted.

On the other hand, some early Eastern Islamic contributions, later forgotten in the Islamic East, were transmitted to Spain and thence to Europe; they have been considered European developments because evidence to the contrary has seemed to be lacking. A good example is the horary quadrant with movable cursor (the so-called *quadrans vetus*), which was invented in Baghdad in the ninth century and (at least in the version with the cursor) virtually forgotten in the Islamic East thereafter; it came to be the favourite quadrant in medieval Europe. What, if any, astronomical knowledge was transmitted through Islamic Sicily remains a mystery, and nothing of consequence is known to have been learned about the subject by the Crusaders.

In the European Renaissance there was no access to the latest Islamic works. So the Europeans contented themselves with new editions of the ancient Greek works, with occasional, almost nostalgic, references to Albategnius (al-Battānī), Azarquiel (Ibn al-Zarqāllu), Alpetragius (al-Biṭrūjī) and the like. A few technical terms derived from the Arabic, such as alidade, azimuth, almucantar, nadir, saphea, and zenith, and a few star-names such as Aldebaran, Algol, Altair and Vega, survived. When the Europeans did come to learn of some of the major Islamic works and to try to come to terms with them it was as orientalists and historians of astronomy, for by this time the Islamic materials other than observation accounts were of historical rather than scientific interest. Thanks to orientalists such as the Sédillots in Paris, works that had been completely unknown to Europeans and mainly forgotten by Muslims were published, translated and analysed. Islamic astronomy was highly respected by such scholars and others, like the historian of astronomy J.-B. Delambre, who, innocent of Arabic, took the trouble to read what his colleagues had written about the subject. But Islamic astronomy, indeed Islamic science in general, received a blow beneath the belt from P. Duhem, a physicist and philosopher ignorant of Arabic, who simply ignored what scholars such as the Sédillots had written. His thesis, that the Arabs were incapable of scientific thought and that whatever merits their science may have had were due to the intellectually superior Greeks, still has many followers, but only amongst those totally ignorant of the research of the past 150 years.

In the period after *c.* 1500 Islamic astronomy declined. All of the problems had been solved, some many times over. Much of the innovative activity had led into a cul-de-sac, from which it would not emerge until modern times, thanks to investigations of manuscripts and instruments. Not that interest in astronomy died down. From Morocco to India the same old texts were copied and studied, recopied and restudied, usually different texts in each of the main regions. But there was no new input of any consequence. Astronomy continued to be used as the handmaiden of astrology, and for the regulation of the calendar and the prayer-times. Where there appears to have been some innovation – such as, for example, in the remarkable device made in Isfahan *c.* 1700 that correctly displays the direction and distance of Mecca for any locality – one can be confident of the existence of an earlier tradition. True some European '*zīj*es', notably those of Cassini and Lalande, were translated into Turkish and their tables adapted for the longitudes of Istanbul and later Damascus. But the old traditions died hard, and Muslim astronomers for several centuries spent more time copying old treatises and tables than compiling new ones.

Conclusion

The reader will have seen that during the millennium beginning *c.* 750 and especially in the period up to *c.* 1050, although also in the period up to *c.* 1500, Muslim astronomers did first-rate work, most of which was not known in medieval Europe at all. Those few Islamic works from the early period that were transmitted, notably the *Zīj*es of al-Khwārizmī and al-Battānī (especially through the *Toledan Tables*) and the banal summary of the *Almagest* by al-Farghānī, convey only an impression of classical astronomy in Arabic garb. But they were in no way representative of contemporary Islamic astronomy in the East, and whilst the Europeans laboured for centuries to come to terms with them, Muslim astronomers were making substantial contributions to their subject that have only been revealed by modern scholarship.

Anyone who leafs through the pages of the two volumes of Fuat Sezgin's *Geschichte des arabischen Schrifttums* listing the manuscript sources for Islamic astronomy and astrology up to *c.* AD 1050, or my survey of the 2,500 scientific manuscripts in the Egyptian National Library, which serves mainly the period after that, will observe the wealth of material relating to this subject that remains untouched by modern scholarship. Very few Islamic astronomical works have been published or have received the attention that they merit. Three out of close to 200 Islamic *zīj*es have been published in the optimum way (text, translation and commentary). We have no published edition of the Arabic versions of the *Almagest* (except for the star catalogue), or of any Arabic recensions or commentaries. Many of the published Arabic scientific texts were printed in Hyderabad, most with no critical apparatus. In view of the lack of a published corpus of texts and the improbability that such a corpus will materialise, there is an obvious need for reproduction of manuscripts of particular importance. Otherwise the historian of Islamic astronomy will be forced to continue to rely mainly on microfilms of manuscripts, which some libraries are unable or unwilling to supply. Likewise most of the historically important Islamic astronomical instruments are still unpublished, although the catalogue currently in preparation in Frankfurt promises to make them better known.

In 1845 L. A. Sédillot, whose privilege it was to have access to the rich collection of Arabic and Persian scientific manuscripts in the Bibliothèque Nationale in Paris, wrote: 'Each day brings some new discovery and illustrates the extreme importance of a thorough study of the manuscripts of the East.' Sédillot, like his father before him but like few historians of Islamic science since, also realised the importance of Islamic astronomical instruments. Given the vast amount of manuscripts and instruments now available in libraries and museums elsewhere in Europe, the United States and the Near East, and the rather small number of people currently working in this field, Sédillot's statement is no less true now than it was a century and a half ago.

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