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Decoding and Demystifying Da Vinci

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DECODING AND DEMYSTIFYING DA VINCI

There has been a great deal of interest in a recent book called The Da Vinci Code. In this article Dr Zohor Shanan Idrisi traces some of the religious symbolism connected to ideas in this book and used from ancient Mesopotamia through to the current day and looks at the ways in which the mathematics involved developed through Muslim contributions.

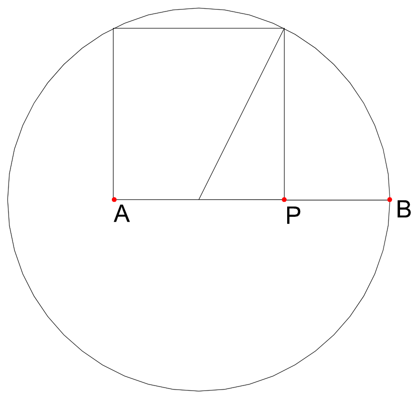
In man's eternal quest to understand his purpose in the universe, much effort and time was spent looking at complex and mysterious things in mathematics, geometry and music to try and find connections to and reflections of the divine. Consequently patterns and symbols were embedded in art as a constant reminder of these ideas. The Mesopotamians were the first to devise a system of proportion. The oldest pentagram originates in ancient Mesopotamia (Iraq) around 3000 B.C. It was referred to as the heavenly body or the "Star". The development of geometry was also important to the Ancient Egyptians to survey their irrigation system and for division of land, since every year it was flooded and the land markings were lost. In their religion their god Osiris was reflected in the waters and his consort Isis epitomised earth. Therefore the prospect of finding a divine strategy to establish the sacredness of this process became a necessity.

The desire to manifest the sacredness of man and his role in the universe, to show that within him there is the cosmos and that he reflects it, drove him to build temples as sanctuaries for the souls of the dead. The human body of the ruler was to rest in the temple for the sake of its soul. The architectural theme of the Centrality of Creation was primordial for the Mesopotamian. Man was formed at the "navel of the earth" where the bond of Heaven and Earth is located. The calculation is determined on a human figure in a circle of which the navel is the centre. Hence the translation into Greek "Omphalos", meaning navel. The designs of such buildings reflect not only the know-how of a nation but also its religious conviction. The equation of the body with a temple is thus an ancient Near Eastern and Ancient Egyptian concept.

Despite the absence of a proper numerical system, the Ancient Near Eastern and Ancient Egyptians were able to divide lengths into simple fractions using a rope-folding method. Although they were confined to drawing their ground plans with ropes and pegs, they had nevertheless managed to trace squares and rectangles correctly by using a grid of intersecting circles; the circles being the one form that can easily be traced with a rope and two pegs. The accurate dimensions of their temples and monuments provide evidence for this. They had also discovered that there existed certain right-angled triangles whose sides were in the proportion of whole numbers, in particular the 3:4:5 triangle. This appears to have been considered as sacred and was known by the Ancient Egyptians as "The Triangle of Osiris". It symbolized the *Tuat* (Underworld).

Irrational numbers, particularly $\sqrt{2}$ and $\sqrt{5}$ can be identified in some of their temple designs. It seems that, in the absence of a numeric system, these were obtained by constructing dynamic rectangles. The association with cosmological ideas and themes of creation and fertility gives ample evidence that they were copying the proportions found in nature. This has been evidenced by applying modern computer-aided design techniques to the structures of the Ancient Egyptian temple of Sesotris I at Tod (XIIth Dynasty c.1950 B.C.) and the Tomb of Rameses IV (XXnd Dynasty c.1140 B.C.). Their geometric system appears to be based on square grids, which shows that their monuments were built using a reproducible geometric method based on squares, circles, polygons etc. Furthermore they employed fractions as a means of computing differences. (Issam Es Said/ Ayse Parman 1976). Finally, in many of their monuments, there is clear evidence of proportions governed by the Golden Ratio Φ.

In its simplest form the Golden Ratio is obtained by a geometric construct based on a square, the base of which (AP) is bisected. The mid-point thus obtained is joined to the opposing corner of the square. Using the line thus produced as radius and the mid-point of AP as centre, an arc is drawn that cuts the extension of the base at B. The Golden Ratio thus obtained is expressed by both AP/PB and AB/AP. Given that AP is unity, the ratio AB/AP can be expressed as ½(1 + √5), which gives a value of 1.618.



Geometric Illustration of AB/AP =Φ

Ancient Egyptian and Babylonian mathematics were transmitted to the Greeks, (the Golden Rectangle was used for the Parthenon, 400 B.C.). Later Pythagoras, who wrote on the Golden mean, referring to it as "*analogia*", received his information from the Egyptian priests. Polycleitus, (c. 450/470 B.C.) was a well-known Greek architect and sculptor. His usage of the mathematical proportions of the human body is reflected in his statues: *Doryphorus*, (Naples Museum) and *Diadumenus* (Athens Museum) and *the Amazon* (New York Museum). The Roman architect and engineer Marcus Vitruvius Pollio (c.70-25 B.C.), wrote his Roman Canon "*De Architectura*" based on earlier Greek treatises including those of Polycleitus. The idea of a circle with the navel of a human figure as its centre is firmly an ancient esoteric and religious concept. Vitruvius relied upon the ancient sexagesimal system of calculation from which is derived our degree of sixty minutes and our minute of sixty seconds within a three hundred and sixty degree circle. This system of mensuration was bequeathed from Ancient Mesopotamia and Ancient Egypt to the Greeks.

Following the collapse of the Roman Empire at the beginning of the 5th century man's concern was primarily focussed upon security and stability, whilst art and science were of necessity neglected. For two hundred years all progress stagnated in the wake of barbarian invasions and the resulting lack of maintenance of public works, such as dams, aqueducts and bridges. With the advent of Islam in the 7th century a new type of society emerged, which quickly established its supremacy and its constructive identity in large sections of the known world. The citizen, whether Muslim or not, soon became confident in the future stability of his environment, so that trade not only reached its previous levels but also began to expand.

In an empire that stretched from the Pyrenees to India, security of communications was vital. The resultant priority given to safety of travel provided a stimulus to trade. There followed a rapid expansion of commerce in which the economic strengths of the Sassanid, Byzantine, Syrian and western Mediterranean areas were united. The establishment of an efficient fiscal system meant that the state could now invest in large public works projects: mosques, madrasas, public baths, palaces, markets and hospitals. Princes and merchants became patrons of intellectual and scientific development. *Waqf* (trusts) were created to provide better education. This sponsorship engendered a creative enthusiasm and a flowering of scientific works and scholarly research. The world in effect became greater as mathematicians, geographers, astronomers and philosophers all contributed to a gradual but definite extension of the horizons of man's existence. The dividend of all this expenditure on learning made an immense contribution to the sum of the increase in man's scientific knowledge that occurred between the 9^{th} and the 16^{th} centuries.

Foremost in the achievements of Muslim scholars was the treatment of numbers. It is impossible to conceive how science could have advanced without a sensible logical numeric system to replace the clumsy numerals of the Roman Empire. Fortunately, by the 9^{th} century the Muslim world was using the Arabic system of numerals, an adaptation of the Hindu system, but with the essential addition of the zero. Without the latter, it was impossible to know what power of ten accompanied each digit. Hence 2 3 might mean 23, 230 or 203. The introduction of this numeric system with its zero was thus the 'sesame' of scientific advancement.

The new numeric system did not only affect science. Its value was manifest in many aspects of daily life, from the calculation of customs dues, taxes, *zakat* (almsgiving) and transport charges, to the complexity of divisions of inheritance. A further useful innovation was the mine of separation in fractions, which eliminated many frustrating confusions.

Islamic civilization produced from roughly 750 CE to 1450 CE a succession of scientists, astronomers, geographers and mathematicians from the inventor of Algebra to the discoverer of the solution of quadratic equations[1]. The list is far reaching, some are well known whilst others remain anonymous. One of the major advances was contained in the work of Al Khawarizmi, who wrote a mathematical work called *Al Jabr wa Al-Muqabala* (820 CE), from whose title is derived the name "algebra". Amongst the achievements that Al Khawarizmi left to posterity were:

a. Solutions to first and second-degree equations with a single unknown, using both algebraic and geometric methods.
b. A method of algebraic multiplication and division.

Al Khawarizmi defined three kinds of quantities:

a. Simple numbers, such as 5, 17 and 131.
b. The <u>root</u> which is the unknown quantity <u>shay'</u> in Arabic meaning "a thing". However, in translations made in Toledo, (the centre for translation of Arabic books), the absence of a "sh" sound in the Spanish language meant that a suitable letter had to be chosen. The choice fell upon "x", which may well explain why Don Quixote is often pronounced as "Don Quishote".
c. "Wealth" (*mal*) the square of the root (x^2).

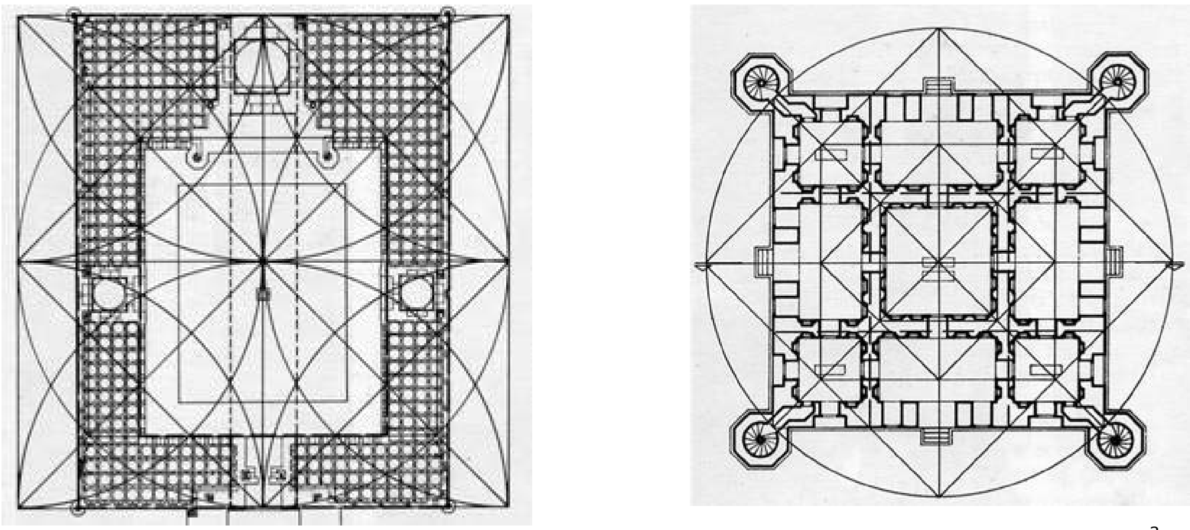
The algebraic equation expressing the Golden Ratio could therefore be written as

$$x:y = (x + y)/x^{*}$$

[1] J.L.Berggren 1986
[*] See Appendix 1 - An algebraic treatment of the golden ratio Φ

Another virtuoso of algebra was Abu Kamil, a 10th century mathematician nicknamed the "Egyptian calculator". He was capable of rationalizing denominators in expressions that involved dealing with powers of x (the unknown) as high as the eighth and solving quadratic equations with irrational numbers as coefficients. Al Biruni (9th/10th centuries) mathematician and physicist, worked out that the earth rotates on its own axis and succeeded in calculating its circumference. Abu Bakr Al Karaji (10th century) is known for his arithmetization of algebra[2].. He also drew the attention of the Muslim world to the intriguing properties of triangular arrays of numbers (Berggren 1983). Al Nasawi (10th century) and Kushyar Ibn Labban worked on problems of the multiplication of two decimals. Subsequently Kushyar explained the arithmetic of decimal addition, subtraction and multiplication and also how to calculate square roots. Abu Al Hassan al Uqlidisi (Damascus 10th century) invented decimal fractions, which proved useful for *qadis* (judges) in inheritance decisions. Al Karkhi (d.1019) found rational solutions to certain equations of a degree higher than two.

Mohamed Al Battani (Baghdad 10th century), mathematician and astronomer, computed sine, tangent and cotangent tables from 0° to 90° with great accuracy. One of his works: *Al Zij* (Astronomical Treatise and Tables), corrected Ptolemy's observations on the motion of the planets. Al Samaw'al Ben Yahya al Maghribi (1171) drew up charts of computations of long division of polynomials; one of the best contributions to the history of mathematics. Ibn Shatir Al Muwaqqit (Damascus 1375 CE) was an astronomer and the timekeeper of the Damascus mosque. His treatise on making astronomical devices and their usage and his book on celestial motions bear great resemblance to the works of Copernicus (1473-1543 CE). Ghiyat al Din al Kashi (1427 CE) raised computational mathematics to new heights with the extraction of fifth roots. He also showed how to express the ratio of the circumference of a circle to its radius as 6.2831853071795865, identical to the modern formula $2\pi r$.



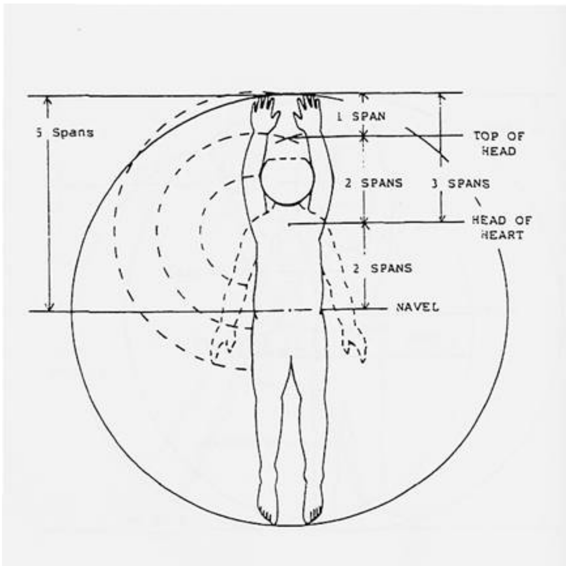
Tomb of Bibi Khanum, Samarqand, 1398 and Tomb of I'timad Ad-dawla, Agra, India, 1678[3]

The potential list of scientists is far too long, so only those connected with the details of this article have been listed. Moving from the realms of science and mathematics to the domain of art; because Islam forbids any kind of human representation, Muslims directed their interest towards abstract forms.

[2] Roshdie Rashed
[3] Issam El Said/ Ayse, Parman, 1976

Meanwhile the concept of centre in a system of proportion remained the focus for the mystics as well as the scientist. As a case in point, the *Ikhwan Safa'* (Brethren of Purity - 10th century CE), encyclopaedists, in their "epistles" (*Rasa'il*), remain the witnesses of a transitional period. They transmitted the knowledge of several civilizations in the literary as well as in the scientific world. The 10th century is a key period in Islamic history, as in the *Ikhwan* can be seen, for the first time in human history, science becoming international on a great scale with the Arabic language as its vehicle. The 10th century was the beginning of a new era of development and extension of knowledge by autonomous research which was in contrast to the previous centuries that were spent in gathering information from different parts of the world, translating them into Arabic and absorbing the knowledge contained therein.

The *Ikhwan* also appear to have been concerned with proportion. They knew of the Roman canon of Vitruvius as a system of proportion, but they considered its proportionality defective as it was centred on the sacrum instead of the navel. In fact the Vitruvian Canon was based on a Greek canon itself based on an Ancient Egyptian canon related to the backbone of the god Osiris. The "sacred backbone" (*Djet pillar*) was a pre-dynastic representation of *Osiris*. It represented stability, endurance and rectitude. ("*os sacrum*") (Barbara Watterson 1984).



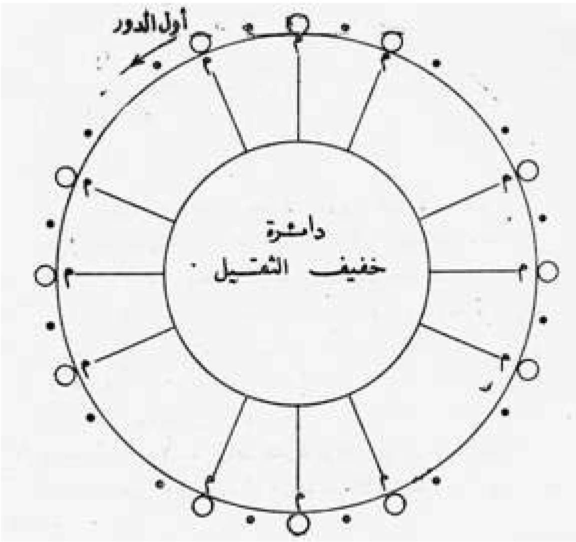
Ikhwan Safa' 10th Century- Infant's proportion

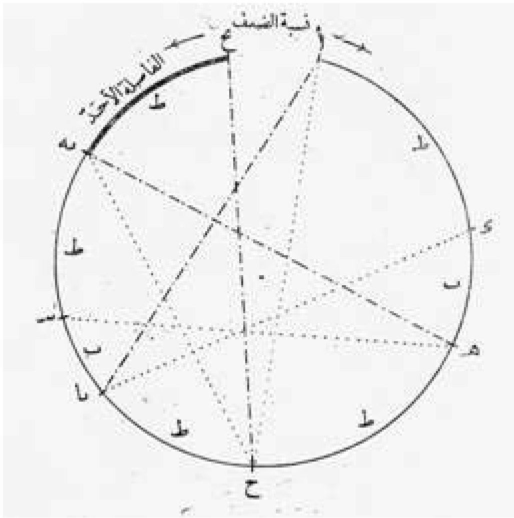
By contrast the *Ikhwan's* epistles show that, after painstaking research, they were able to establish the centrality of the human body with the navel; the mid point on the centre of the circle with both superior and inferior members extended, i.e. with fingertips and the tips of the toes touching the circumference. Simply by using the body of an infant they achieved the ideal proportion required. In fact, the infant's navel begins to be disproportionately placed after the age of seven; the age of innocence. Hence before the age of seven appropriate measurements were obtained. At birth the mid point of the body is at the navel. As the individual grows the mid point drops until it reaches the groin (*sacrum*). The proportional ratio produces an ideal figure for religious painting. The width is eight spans, the height is ten spans and the mid point is on the navel.

The division of the figure follows these ratios[4]:

8 heads	=	whole body
1/8 body	=	1 foot
1/8 body	=	1 face
1/3 face	=	1 forehead
4 noses	=	1 face
4 ears	=	1 face

The affinity between natural measurements and mathematical expressions made the whole concept sacred to artists. The centrality of the circle (the Earth) and the centrality of the navel, lieu of life sustenance, demonstrated the theophanic manifestation. The canons of divine proportion were reflected in cosmology, musicology, calligraphy and in all arts from the 10th century CE. The canons of proportion were seen as the key to finding harmony and, for the mystics, closeness to God. The Golden Ratio was found with more frequency in living things, such as mollusc shells, plant-leaves etc.





Kitab al Adwar - Music composition *Urmawi b.*1230

The intellectual dynamism that caused so many advances in science did not leave the arts unaffected in its wake. As regards music, at the beginning of the Islamic medieval period musical theory was at best rudimentary. No doubt this was due to the fact that no real innovations had occurred since the time of Plato and Aristotle. The Neo-Pythagorean philosopher, Nichomachus of Gerasa (2nd century CE), who associated music with mathematics, was unable to develop his ideas beyond associating the numbers from one to ten with gods and goddesses. The Byzantine music theorist Michael Psellus (1018-1080 CE) wrote, "The kind of music that occupies minds today is only a faint echo of the Hellenistic music"[5] . Melodic interpretation was seen by the ancients, (the Mesopotamians and the Greeks), as circles representing the seven planets, corresponding to the relationship of the human soul, in terms of its emotional harmony, with the cosmos.

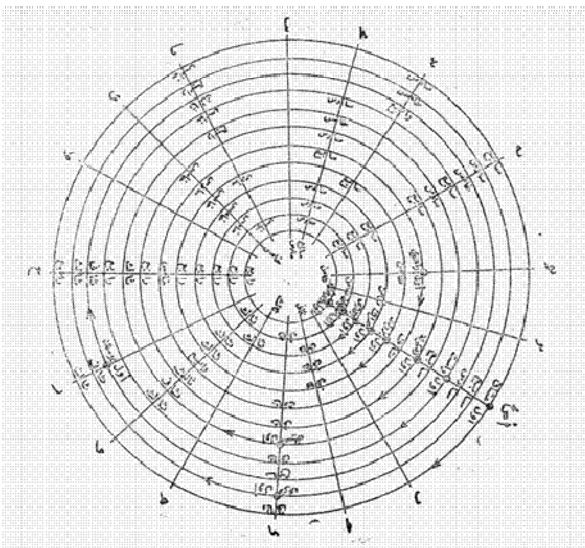
[4] Fakour Mehrdad 1993
[5] Egon Wellesz 1961

The old Greek considerations of modal ethos did not inspire the Muslims. They preferred to focus their compositions upon astronomy, cosmology and the environmental atmosphere. The Greek scale was a sequence of descending sounds. This concept was effectively turned upside down by Muslim scholars, who instituted an ascending scale. The Muslim desire to organize, rationalize and measure was now directed towards music. Perhaps because of possible links with cosmology, they chose to apply their progress in mathematics to the theory of music. Thus mathematics became a basic feature of musical composition and music was treated as an intellectual subject. Muslim scholars revolutionized musical theory, both in form and content, evolving a notational system with 24 quarter-steps per octave.

The theory of harmony of heavenly bodies rested on the movement of the planets and the laws of mathematics governed the relations of consonance. In systemizing such relationships the Muslim scholars ensured that the ratios of the various notational intervals were expressed in precise mathematical terms. The theorists interpreted the universal order as the relation between musical harmony and the mathematical proportions that it formed. Numbers in accordance with the ratios 1/2, 2/3 and 3/4 were used. Thus a real mosaic of meticulous calculations of sonic intervals was created for the first time. As a logical consequence this also involved the use of the Golden Ratio.

This form of interpretation was perpetuated by a large number of scholars writing treatises over several centuries:

Al Kindi	(874)	Ibn Bajja	(d.1139)
Al Farabi	(d.950)	Ibn Tafayl	(d.1185)
Ibn Sina	(d.1037)	Al Urmawi	(1275)
Ibn Zaylad	(d.1048)	Ibn Al Khattib	(14th century)
Al Biruni	(1050)	Al Ladhiqi	(16th century)
Ibn Nadhim	(10th century)		



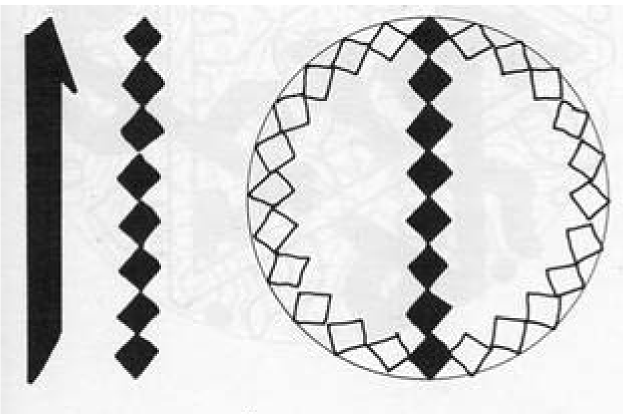
Kitab al Adwar- Safi Yudin Abdel Mum'in Ibn Yusuf Ibn Fakhri Al Urmawi B.1230

Muslim classical music, (*maqamat, nuba* and *'ala*), is still vivid, although a great deal of the repertoire has been lost. It is still appreciated by its enlightened supporters, who fervently believe in its therapeutic qualities. Certainly it succeeded in sparing the life of Al Urmawi during the sack of Baghdad (1258 CE), when its quality had such an effect upon the bloodthirsty Mongol Hulagu. Not only did Al Urmawi's lute save his life, but also it gained him a pension of 10,000 dinars! No doubt because of the language problem, but also because of a general lack of mathematical and scientific knowledge, Europeans had little understanding of medieval Muslim music. Jerome of Moravia (13th century) struggled unsuccessfully to understand Al Farabi as did Michael the Scot (d.1235). The following passage from Farmer's (1934) translation of Al Farabi gives an idea of what the medieval Europeans missed:

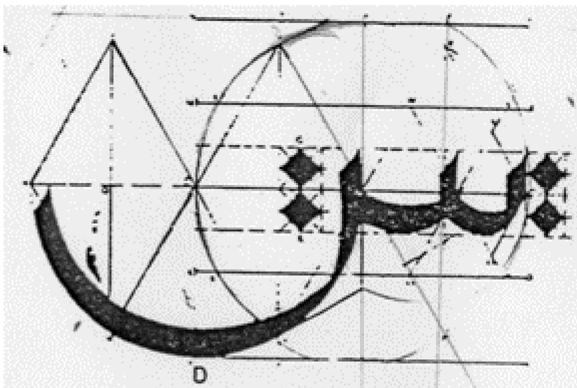
> *"To this science are three roots – metre, melody and gesture. Metre devised to regulate a rational comprehension of diction. Melody was devised to regulate the parts of acuteness and gravity [in sound], and to it two roots have been included in the sense of hearing. Gesture has been included in the sense of seeing which, by coincident motions and corresponding proportions, has been arranged to agree with metre and sound. This art, therefore, is included in two particular senses – hearing and seeing."*

The above is only an outline of the matter. A full analysis of the musical methodology of the Islamic medieval period would be too vast a subject for the scope of this article.

The desire to rationalize and organize all aspects of artistic endeavour was extended to the field of calligraphy. In the early 10th century there were many forms of cursive writing, but they were all lacking in style. Ali Ibn Muqla (d.940 CE) devised a canon for the cursive style based on a precise complex geometrical and mathematical system based upon the circle, a standard *alif* and a rhombic dot. The rhombic dot was dependent for its size on the size of the reed pen used for writing. The standard *alif* was dimensionally equal to a vertical array of eight of such rhombic dots. This was also the set diameter of the guiding circle. Thus the curved letter "*sin*" had its three curves written across the diameter with widths of one dot, two dots and four dots.



Ibn Muqlah d.939. The proportion of a diameter to the circumference of its circle is 1 to π



Calligraphy Fakour Mehrdas 1998

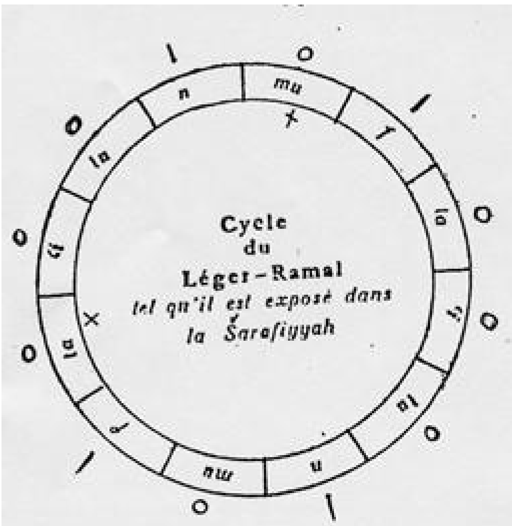
Needless to say, before reaching calligraphy, the rationalization had made its mark on the structure of Arab poetry. In the phonetic pattern of the Arabic language, there are three short vowels called *harakat* (a, o and i) and three *madd* letters, (long vowels), *alif, waw* and *ya*. The latter are called consonants, because they have a special function in the *Taf'ila* system (poetry). The consonants, when they have a " ° " (*sukun* = 'silence') above them, have no following vowel sound.

Al Khalil Ibn Ahmed (b.Baghdad 718 CE), a philologist, categorized the metres of classical Arabic poetry into sixteen distinct types (*Taf'ila*) of eight-word units. A system of scanning was used, where " **1** " indicates a sound and " **0** "indicates silence (i.e. no vowel). The verbal metre is a coded order repeated only in one or two units on the principle of circles called *'arud*.

Example of a *Tawil* metre:

Type	sub-units
fa'ulun	**1 1 0 1 0**
mafa'ulun	**1 1 0 1 0 1 0**
fa'ulun mafa'ulun) *fa'ulun mafa'ulun*)	**1 1 0 1 0 1 1 0 1 0 1 0** x **1 1 0 1 0 1 1 0 1 0 1 0**

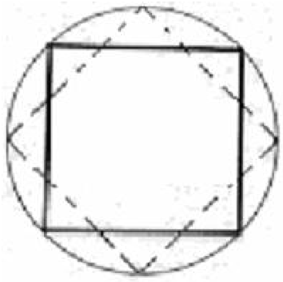
The ratios of sounds to silence signs in the whole, the ½ and the ¼ of the *Tawil* metre are 28:20, 14:10 and 7:5.



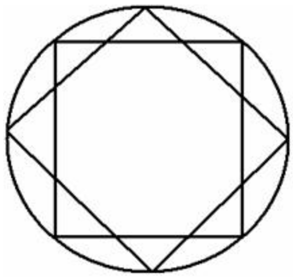
Baron d'Erlanger

√2 system of proportion:

The verbal metres can be expressed geometrically. Thus two successive verbal units repeated four times (e.g. *Tawil* or *Basit*) can be illustrated by the repeat pattern of two squares within a circle, giving the octagonal star. The *Basit* metre differs only in that the two squares can be in any position except superimposition.



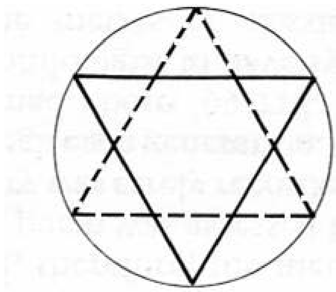
Tawil -An octagonal grid Mutaqarib metre



Basit - An octagonal grid *Mutadarik* metre

Hexagonal repeat unit with √3 system of proportion:

Where there is one verbal unit repeated six times (e.g.*Ramal*), or two units repeated three times (e.g. *Kamil*), the pattern is that of a fixed hexagonal star (*Ramal*) or two triangles forming the latter, but capable of being in any position other than superimposition.



Hexagonal *Kamil metre*

In parallel and for the sake of comparison, in the European Middle Ages, Leonardo Fibonacci (1170-1250 CE) was the prominent mathematical scholar of the 13th century. He is said to have been educated in Bugia, (present day Bejaia in Algeria), and possibly tutored by Arab mathematicians. He is stated to have been born in Pisa and to have returned there after thirty years. He is credited with the authorship of many mathematical texts and the creation of the Fibonacci sequence. He attracted the interest of the Holy Roman Emperor, Frederick II, who invited him to his court. He is also described as having translated and deciphered Arabic texts.

An analysis of the above information shows that there are major flaws in the Fibonacci story as related. Firstly a long sojourn by an Italian in Bugia, Algeria in the late 12th century was a virtual impossibility. The first crusade had been launched in 1099 CE. and by the time of Fibonacci's birth the eighth was already under way. Furthermore Bugia lay within the territory of the ultra-conservative Al Mohad dynasty, where there was no room even for Ibn 'Arabi, who fled North Africa because of his controversial beliefs. Even if a foreigner had been allowed to reside in the town, the place for advanced education was the mosque and its madrasa!

The suggestion that an Italian could acquire sufficient knowledge of Arabic to read and understand mathematical works also seems unlikely. The Arabic ten-digit numeric system is said to have reached Europe during the 12th century through translations of Al Khawarizmi's *Algebra*. However this can not be taken as an indication that the system was widely used. On the contrary there was stubborn resistance to its use and a tendency to stick to clumsy Roman numerals.

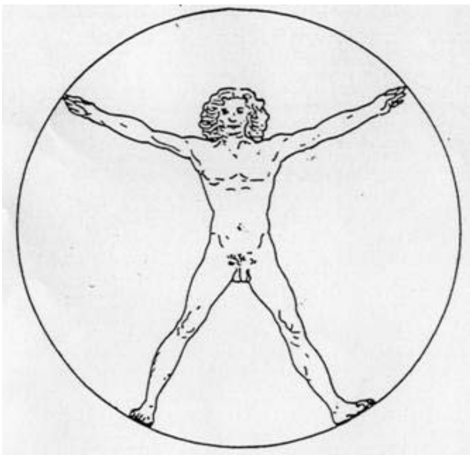
Added to the rarity of numeric skills was the fact that the Arabic word for mathematics *riyaddiyyat* had no equivalent in Latin and was thus not properly understood. The two Latin-Arab glossaries that have survived both translate *riyaddiyyat* by confusing it with *riyada = domat,* meaning "sport", (*Latino arabicum glossarium* (11th century) and *Schiaparelli vocabulisto in arabico* (13th century)). It is on record that Michael the Scot, a contemporary Arabic scholar in 1235 CE was still writing about "*domatrix*". Despite such disadvantages the Italian Fibonacci was writing mathematical examples that plagiarized the work of the Arab mathematician Abu Kamil (1000 CE), in the same manner that Al Karaji's work on triangular numeric arrays re-emerged in 1665 as Blaise Pascal's Triangle.

However, this analysis does not diminish Fibonacci's mathematical ability, but it does cast grave doubts about his nationality. A far simpler and more logical explanation of the story is that Fibonacci was born a Muslim and received a mathematical education in a scholastic environment vastly superior to that available in contemporary Italy and Europe. The Golden Ratio Φ could only belong to the original environment in which he had been educated, as did the ability to evolve the Fibonacci series from earlier observations in nature and the solution to his 'rabbit problem', plus the indication that the latter produced the "Golden string".

This leads to the logical conclusion that Fibonacci was perhaps another case of "brain-stealing", similar to that of Hassan Al Wazan alias Leo Africanus. This North African pilgrim was kidnapped on his way to Mecca for the *Hajj* in the 15th century and retained to teach Arabic and translate texts for the Pope.

Da Vinci (b.1452 CE Tuscany) was an Italian painter, sculptor and architect, but was also depicted by the Italians as a musician, an engineer, a mathematician and a scientist. The secular commercial rivals of the papacy, the Medici family, commissioned him to carry out works of art on religious themes. However, it was noticed that he had incorporated controversial androgynous elements in his paintings. The Church took deep offence at the manner in which the Medici commission had been executed and Da Vinci was accused of spiritual hypocrisy and of sinning against God.

The Renaissance Humanists had fuelled interest in Graeco-Roman classicism in the 15th century CE. One of their scholars, Luca Pacioli wrote a treatise entitled "Divina Proportiona". Thus the concept of sacred geometry had finally reached the world of the humanists. With Pacioli, Leonardo Da Vinci produced a drawing called "The Vitruvian Man", based upon the Roman Canon of Vitruvius. As mentioned above this Canon had been analysed by the *Ikhwan Safa'* and was considered by them to be proportionally defective and it was these proportions that Da Vinci used. His other works, in particular, the paintings "Mona Lisa", "Madonna of the Rocks" and "The Last Supper" were all cast on the "Divine Concept of Proportion", the Golden Ratio.



Vitruvian man-flat feet on the circle

The collection of books, whether scientific or artistic, the gathering of information, plus the central power of politics, were all the prerogative of the Church. Da Vinci's mentor, Luca Pacioli (1445-1517) was in fact a Franciscan friar. He wrote *summa de arithmetica, geometria, proportioni et proportionalita*". This work gives a summary of the mathematics known at the time. However, it contained nothing that was original, since its content was clearly based on Muslim works of previous centuries. Its importance lies in the fact that, on publication in Venice in 1494, it became one of the earliest printed mathematical books. In Italy there was a general interest in the concept of divine proportions and it was in fact Pacioli that introduced Da Vinci to the Golden Ratio. The latter found the ratio aesthetically satisfying both from a mathematical and artistic point of view.

To conclude, the Muslim culture brought to the world a great surge of attainment. At first it acted as a compiler of knowledge of everything in the world of science and art. It then spread it through its lands without prejudice of colour or race. All contributed to this Muslim effervescence that was manifest from India to Africa and to southern Europe. This universality cannot be denied and the Arabic language became the *lingua franca* for all knowledge. The Arabic numeric system is one of man's greatest achievements. It should be seen as a blessing from heaven, since it dragged mathematics from stagnation to progress. It is tempting to suggest that the development of mathematics should be more valued than the discovery of the skill of writing. In short, it was a great liberating force.

The ancient civilizations discovered the Golden Ratio and used it exclusively in buildings with a sacred or divine connection. For each successive civilization the Golden Ratio served as an ontological union with the Creator. The Muslims with their superior mathematics had a deep understanding of the Golden Ratio and they used it in every conceivable type of structure and in their abstract art forms. In every place in which they have lived, it is a hallmark of their presence.

In Medieval Europe the Church was at one and the same time the patron and the filterer of knowledge. By stubbornly refusing the transmission of knowledge and information from the Muslim lands to the Christian world, it deprived its flock of scientific progress for several centuries. Thus the Arabic numeric system and Algebra both took three centuries to reach Europe, decimal fraction operations took five centuries to be taken into general use and it took six centuries to learn that the earth rotated on its own axis. Without such impediments it would have been possible for Europe to make the quantum leap in scientific knowledge from its state at the end of the Roman Empire to the age of Newton at least three centuries earlier.

Returning to Da Vinci and the Golden Ratio, the Medici's interest in "divine" aesthetics prompted them to use Da Vinci. The latter brought no new discoveries to science, other than employing the ratio that he had learned from his mathematical mentor Pacioli.

Several parameters have been used in this article; science, mathematics, calligraphy, poetry and music to illustrate the milieu in which so much progress was achieved. It is clear that in the Muslim environment, the Golden Ratio was frequently present, well known and understood and it was judged in terms of beauty and symmetry. However, in keeping with the sober theology of Muslims it had no mystically divine nature.

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Appendix - An algebraic treatment of the golden ratio Φ

by Lamaan Ball

To show the algebraic equations behind the golden ratio Φ, we start by showing the generic solution for a quadratic equation:

(1) $$ax^2 + bx + c = 0$$

(2) $$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

(3) $$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

(4) $$\frac{-b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}} = x$$

(5) $$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x$$

To get the ratio Φ we want the value $\frac{1}{2}\left(1 + \sqrt{5}\right)$ to be a solution to a quadratic.

To match this equation b must equal -a, to easily produce the ½ and the square root, then we have:

(6) $$a^2 - 4ac = 5$$

Any value will work here, so we can pick a simple one to find an equation that works.

Let a = 1, then c = -1 as a simple case.
Putting this back into (1) we get an equation which results in Φ as a solution:

(7) $$x^2 - x - 1 = 0$$

Verifying this quadratic equation we put the values a=1, b=-1,c=-1 into (5) and confirm that we see the solution we wanted:

(8) $$\frac{-(-1) \pm \sqrt{(-1)^2 - (4 \times 1 \times -1)}}{2 \times 1} = x$$

(9) $$\frac{1 \pm \sqrt{5}}{2} = x = \Phi$$

Equation (7) can also be expressed as

(10) $$\Phi = \frac{1+\Phi}{\Phi} = 1 + \frac{1}{\Phi}$$

This also works as an iteration such that

(11) $$\Phi_{new} = 1 + \frac{1}{\Phi_{old}}$$

Starting with practically any value for Φ_{old} within 10 iterations Φ is reached to within four significant figures. Within 20 iterations Φ is reached to within 8 significant figures (1.6180339…). If we rewrite Φ as a ratio of x and y this may be expressed in the following way:

(12) $$\frac{x}{y} = 1 + \frac{y}{x} = \frac{x+y}{x}$$

This can also be expressed as an iteration with

(13) $$y_{new} = x_{old}$$

and

(14) $$x_{new} = x_{old} + y_{old}\,.$$

We can start with any values of x and y (other than x=0 and y=0) and following this iteration, we will, within a few iterations, have reached an accurate value of Φ. If we take the simplest case of x=1 and y=1 as our initial values the series of values for either x or y is the famous "Fibonnaci series".

This was expressed by Fibonacci as the "rabbit" problem which gives specific meanings to x and y. In this expression x is the number of adult rabbit pairs and y is the number of young rabbit pairs. Each cycle of iteration a new pair of young rabbits is born to each adult pair. This corresponds exactly to (13). Each cycle the young rabbit pairs mature adding to the total of adult rabbit pairs. This corresponds exactly to (14).

Iteration	1	2	3	4	5	6	7	8	9	10
x_{old}	1	2	3	5	8	13	21	34	55	89
y_{old}	1	1	2	3	5	8	13	21	34	55
x_{new}	2	3	5	8	13	21	34	55	89	144
y_{new}	1	2	3	5	8	13	21	34	55	89

144 / 89 = 1.6179775280898876404494382022472 $\approx \Phi$