



# The Arabic Sources of Jordanus de Nemore

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# THE ARABIC SOURCES OF JORDANUS DE NEMORE

Prof. Menso Folkerts and Prof. Richard Lorch\*

## 1. Jordanus de Nemore: Life and Works<sup>1</sup>

### 1.1. Life

Historians of mediaeval mathematics agree that Jordanus de Nemore was one of the most important writers on mechanics and mathematics in the Latin West – to be compared only with Leonardo Fibonacci and Nicole Oresme. But almost nothing is known about his life. He must have lived before the middle of the thirteenth century, because Jordanus' works are mentioned in Richard de Fournival's *Biblionomia*, a catalogue of books compiled towards 1250, and because Campanus cites Jordanus in his redaction of Euclid's *Elements* which must have been written before 1259.<sup>2</sup> Dr. Busard assumes that there was a close relationship between Jordanus and Richard de Fournival, because Richard's *Biblionomia* contains not only most of the genuine works of Jordanus, but also most of the sources that Jordanus used for his treatises.<sup>3</sup> It is also remarkable that nearly half of the manuscripts which transmit one of the most important writings of Jordanus, his *Arithmetica*, were written or are extant in Paris. Therefore it is possible that Jordanus lived and taught at Paris in the first half of the thirteenth century and it may be assumed, as Jens Høyrup has, that there existed in Paris until about 1250 a "Jordanian circle" which was influenced by Jordanus himself.<sup>4</sup>

Beginning with an article by Maximilian Curtze in 1887,<sup>5</sup> Jordanus has sometimes been linked with the University of Toulouse, but Ron B. Thomson has given convincing arguments that there is no reason at all to associate Jordanus with that university.<sup>6</sup> The question has been discussed whether Jordanus de Nemore might be identical with Jordanus of Saxony, the second Master-General of the Dominican order from 1222 to 1237,<sup>7</sup> since P. Treutlein drew attention (in 1879) to a statement by the fourteenth century chronicler Nicholas Trivet.<sup>8</sup> Trivet wrote that the Master-General Jordanus was "by nationality a Teuton from the diocese of Mainz" and that "reputed to have been outstanding at Paris in the secular sciences, especially mathematics, he is said to have written two very useful books, one *De ponderibus* and another *De lineis datis*". While Treutlein, Curtze and Cantor agreed that the Master-General Jordanus of Saxony and Jordanus de Nemore were one person, the historian of the Dominican order, H. Denifle (in 1887) – and in this century Marshall Clagett (in 1984) – disagreed, mainly, because the name 'de Nemore' never appears in the writings of Jordanus of Saxony nor in any source pertaining to him, and because the writings of Jordanus of Saxony show no special interest in mathematics. Two of the historians of mathematics who have done

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<sup>1</sup> Published originally in *Etudes d'histoire des sciences arabes*, edited by Mohammed Abattouy, Casablanca, 2007, pp. 121-139.

<sup>2</sup> In this year the earliest known manuscript (Florence, BN, Magliabecch. XI 112) was written. Campanus mentions Jordanus in his commentary to *The Elements* V. def. 16.

<sup>3</sup> Busard 1992, pp. 121-122.

<sup>4</sup> Høyrup 1988, p. 351.

<sup>5</sup> Curtze 1887, p. vi.

<sup>6</sup> Thomson 1974.

<sup>7</sup> E.g. in Grant 1973, pp. 171-172, and in Thomson 1978, pp. 10-17.

<sup>8</sup> Treutlein 1879, p. 129.

research on Jordanus de Nemore in modern times, Thomson (1978) and Busard (1991), agree that it is possible that the two men were identical, although this is neither probable nor improbable.

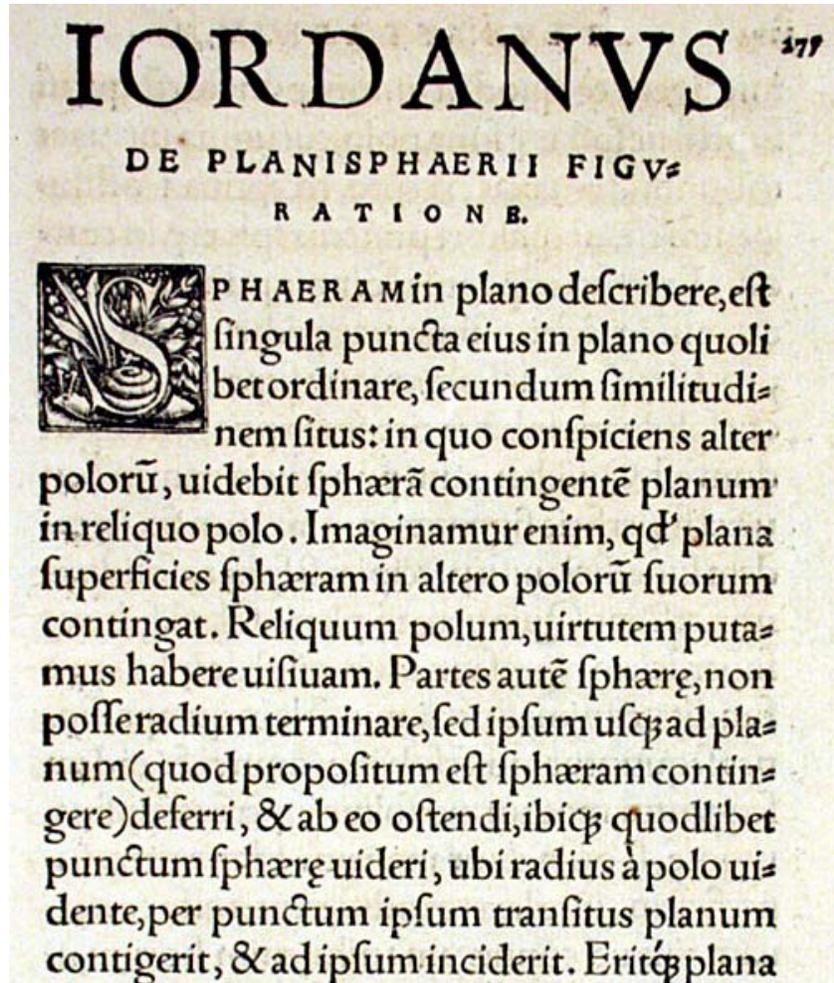


Figure 1: Extract from Jordanus' *De planisphaeri figuratione*. Source:

[http://www.ub.unibas.ch/kadmos/gg/pic/gg0287\\_009\\_txt.htm](http://www.ub.unibas.ch/kadmos/gg/pic/gg0287_009_txt.htm)

## 1.2. Writings

Ron B. Thomson has given an exhaustive list of all manuscripts which contain texts attributed to Jordanus de Nemore.<sup>9</sup> From this it is clear that there are six treatises genuinely ascribed to him (most of them existing in more than one version):

<sup>9</sup> Thomson 1976.

### 1.2.1. *Liber philotegni*

This treatise, which has been edited by M. Clagett in 1984,<sup>10</sup> appears to be a genuine work by Jordanus and to have been reworked under the name *Liber de triangulis Iordani*.<sup>11</sup> It is an advanced textbook on geometry at a very high level. The *Liber philotegni* may be divided into the four parts that later appeared as separate books in the *De triangulis*. The principle contents are:

- Prop. 1-13: on triangles, and primarily their comparison in terms of angles and sides and lines drawn from angles to sides.
- Prop. 14-25: on the division of triangles, and lemmata for the propositions concerning divisions.
- Prop. 26-37: comparisons of arcal and circular segments cut off by chords, both those within a single circle and those within tangent circles, and extra-circular areas included between tangents and arcs.
- Prop. 38-63: on polygons that are irregular or regular, inscribed or circumscribed, inserted in one another, isoperimetric or not.
- With prop. 46 the "shortened version" ends, but the remaining propositions appear to be an integral part of the original *Liber philotegni*.<sup>12</sup>

Especially interesting additions in the *De triangulis* are solutions to the problems of trisecting an angle and of finding two mean proportionals between two given lines, and also a proof of Hero's theorem on the area of a triangle.

### 1.2.2. *Elementa de ponderibus*

There are some Latin treatises on statics in the manuscripts attributed to Jordanus, in which the dynamical approach of Aristotelian physics is combined with the abstract mathematical physics of Archimedes, the proofs being presented in the Euclidean way. But only one treatise, the *Elementa super demonstrationem ponderum* or *Elementa de ponderibus*, may be definitely assigned to him.<sup>13</sup> This treatise, which contains seven postulates and nine theorems, is significant, because it introduces component forces into statics and the idea of "positional gravity" (*gravitas secundum situm*), and also gives a new proof of the law of the lever. As in the case of the *Liber philotegni*, there is a reworking of the *Elementa*, the *De ratione ponderis*, in forty-five propositions; this might also be attributed to Jordanus. Inter alia, this reworking contains a discussion of weights on inclined planes – for instance, the first known proof of the conditions of equilibrium of unequal weights on planes inclined at different angles.<sup>14</sup>

Both treatises are based upon Greek works that were mostly transmitted through the Arabic and on Arabic works – for instance, the *Liber karastonis* attributed to Thābit ibn Qurra – in the same tradition. It seems that the author (or authors) made use of intermediate Latin commentaries.

### 1.2.3. The *algorismus* treatises

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<sup>10</sup> Clagett 1984, pp. 196-257.

<sup>11</sup> Edited by Clagett 1984, pp. 346-429.

<sup>12</sup> Clagett 1984, p. 174.

<sup>13</sup> On the affiliation of the texts *De ponderibus* see Brown 1967.

<sup>14</sup> See Moody and Clagett 1952, p. 169. The text is edited on pp. 175-227.

There are various *algorismus* treatises ascribed to Jordanus. Although they have not been edited in their entirety, several articles by G. Eneström appeared in the *Bibliotheca mathematica* in 1906-14 on this topic.<sup>15</sup> Eneström published the introductions to these texts and their propositions. From Eneström's research it seems to be clear that there were three different *algorismi*, each of them containing two parts, the first on integers and the second on fractions. While the first two sets<sup>16</sup> might have been written by Jordanus, this is not likely to be true of the third, which is certainly related to the first two.<sup>17</sup> The treatises that seem to have been written by Jordanus teach the six basic operations with integers (including duplication and mediation) and the extraction of the square root within the Arabic number system, but without examples and in a more formal way than in the common *algorismus* treatises of the thirteenth century (Johannes de Sacrobosco, Alexander de Villa Dei). All this is strongly reminiscent of Arabic texts (which begin with al-Khwārizmī's *Arithmetic*). But there is no reason to assume that Jordanus had an Arabic text or a translation from the Arabic before him, because there were many *algorismus* treatises in the West at the end of the twelfth and the beginning of the thirteenth centuries, some of which might have been the source for Jordanus.<sup>18</sup>

#### 1.2.4. *De numeris datis*

In his *De numeris datis*, which has been edited by B. Hughes in 1981,<sup>19</sup> Jordanus solved algebraic problems in a way different from that found in Arabic texts. He formulated problems by saying what is known and what has to be found, and then transformed the initial equation into a canonical form by using letters to represent numbers. At the end of every problem he gives a numerical example. Although some bits and pieces can be found in other works, the whole is not a compilation, but a unique tract in advanced algebra – as Busard puts it,<sup>20</sup> it is the "first advanced algebra to be written in Europe after Diophantus". Hughes has indicated that there are two sets of manuscripts, one containing 95 propositions, the other 113. As for sources, the approach is too different from that of al-Khwārizmī for the latter's *Algebra* to have been the decisive influence – and in general, we have found no telling evidence of any Arabic source for this work.

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<sup>15</sup> Eneström 1906-07, 1907-08, 1912-13, 1913-14a, 1913-14b.

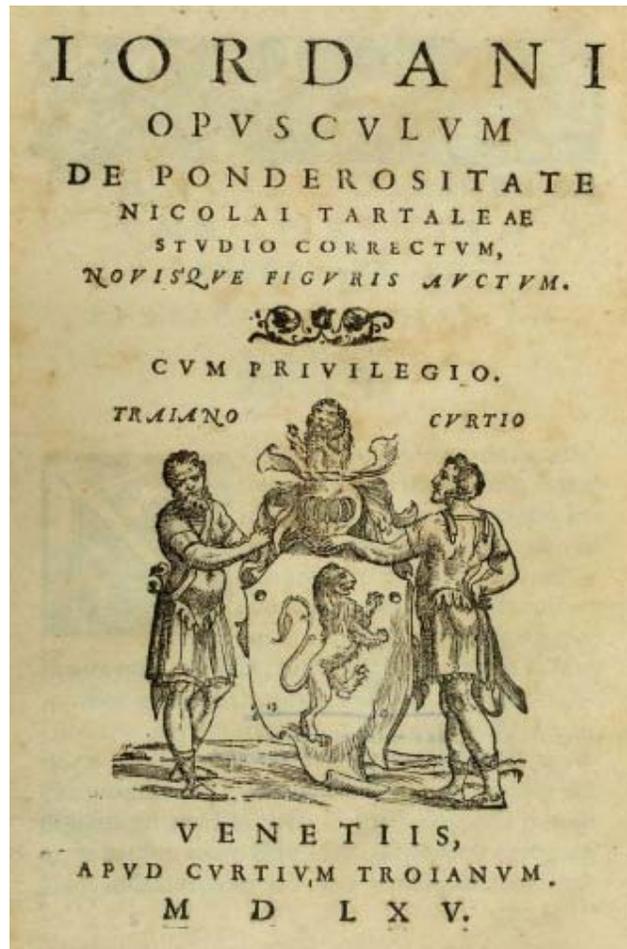
<sup>16</sup> Set 1: *Opus numerorum* (incipit: *Communis et consuetus rerum cursus virtusque*) and the *Tractatus minutiarum*; set 2 (probably later than set 1): *Demonstratio de algorismo* and *Demonstratio de minutis*.

<sup>17</sup> Entitled: *Algorismus demonstratus* (sometimes divided into *Algorismus de integris* and *Algorismus de minutis*). It was published by Johann Schöner (Nuremberg, 1534).

<sup>18</sup> Apart from al-Khwārizmī's *Arithmetic*, which was translated into Latin in the twelfth century, the oldest treatises are the *Liber ysagogarum* and the *Liber alchorismi*, both from the twelfth century. See the editions by Allard 1992.

<sup>19</sup> Hughes 1981.

<sup>20</sup> Busard 1991, p. 10.



**Figure 2:** Jordanus de Nemore, *Liber de ratione ponderis* in the edition of Nicolo Tartaglia (Venice, 1565). Source: <http://archimedes.mpiwg-berlin.mpg.de>.

#### 1.2.5. *De plana spera*

This treatise, which was edited by Ron B. Thomson,<sup>21</sup> may be compared with Ptolemy's *Planisphaerium*. It treats the principles of stereographic projection – the central concept used in constructing the astrolabe, – and gives inter alia a general demonstration of its fundamental property, i.e. that circles are projected as circles. There are three versions, versions 2 and 3 being different expansions of the original text (version 1), which is closest to Jordanus' original.

#### 1.2.6. *De elementis arismetice artis*

Although the *De elementis arismetice artis* was the most widely known mathematical work of Jordanus, it was not edited in its original form until 1991.<sup>22</sup> It is divided into ten books and comprises more than 400 propositions. Similarly to Euclid's *Elements* – and, as it seems, derived from it – Jordanus starts with definitions and postulates and then proceeds to the enunciations. This treatise became the standard book

<sup>21</sup> Thomson 1978.

<sup>22</sup> Busard 1991.

on theoretical arithmetic in the Middle Ages, together with Boethius' *Arithmetica*, which is less formal and more philosophical.

### 1.2.7. Other writings

Under the treatises dubiously ascribed to Jordanus is the *Liber de proportionibus*. Busard, who edited it,<sup>23</sup> was not able to say whether it is an original work or a translation from the Arabic (Thâbit ibn Qurra). If the former, the Arabic influence is very clear.

## 2. Possible Arabic sources of the three principal mathematical works

### 2.1. Liber philotegni

It is not surprising that in the *Liber philotegni* Euclid's *Elements* are mentioned several times, mostly by giving citations as "per ultimam quinti Euclidis."<sup>24</sup> At two places Jordanus gives the text of an enunciation by Euclid: in prop. 18 he cites Euclid V.19 literally,<sup>25</sup> but in a way which differs from the best-known Latin Euclid text, the so-called Version II formerly attributed to Adelard,<sup>26</sup> as well as from the other Euclid texts which originated in the twelfth century (Adelard I, Hermann of Carinthia, Gerard of Cremona). In prop. 28 Euclid III.7 is cited,<sup>27</sup> but here, too, the wording differs somewhat from the common Euclid texts – though it is more similar to Version II and to the Hermann texts than to the translation by Adelard and by Gerard.<sup>28</sup> It seems that Jordanus did not intend to cite these propositions word-for-word. But it is evident that his source was a text that came from the Arabic, because he mentions twice the word *mutekefia* (= reciprocally proportional),<sup>29</sup> which is also given, with the same meaning, in propositions VI.13 and 14 in three of the Arabic-based texts (Adelard I, Hermann, Version II). It should also be mentioned that one of the earliest manuscripts of the *Liber philotegni*<sup>30</sup> cites (prop. 28) the Pythagorean theorem by *per dulk*<sup>31</sup>: the term *dulcarnon* (= the two-horned) for this theorem came from Arabic texts, was first used in the West in some manuscripts of Version II and became later very common.<sup>32</sup>

It seems that Jordanus also used another treatise by Euclid: the *Liber divisionum*. Today this text is available only in Arabic manuscripts. In the twelfth century it was translated into Latin by Gerard of Cremona, but his translation is lost. In propositions 21-23 of the *Liber philotegni* Jordanus presents problems on bisections of triangles, and it is very likely that he used Gerard's translation of Euclid's *Liber divisionum*.<sup>33</sup> The following proposition (24, trisection of a triangle by drawing lines from a point in the triangle to each of the three angles) is not in the extant Arabic text of the *Liber divisionum* of Euclid. But

<sup>23</sup> Edited by Busard 1971.

<sup>24</sup> In props. 9, 19 and 27. Similarly in props. 7, 18, 34.

<sup>25</sup> "Juxta illam quinti Euclidis: si linea ad lineam ut pars ad partem, ergo ut residuum ad residuum": Clagett 1984, p. 212, lines 17-18.

<sup>26</sup> It has: *Si a duobus totis due porciones abscondantur fueritque totum ad totum quantum abscisum ad abscisum, erit reliquum ad reliquum quantum totum ad totum.*

<sup>27</sup> "Juxta illud Euclidis in 3o: si a puncto in diametro preter centrum assignato etc.": Clagett 1984, p. 224, lines 10-11.

<sup>28</sup> Version II and Hermann of Carinthia: *Si in diametro circuli punctus preter (Hermann: super) centrum signetur; Adelard I: Si supra diametrum circuli punctus alius a centro assignatus fuerit...; Gerard of Cremona: Si super diametrum circuli punctum signetur quod sit extra centrum...*

<sup>29</sup> Prop. 9: *nam latera sunt mutekefia*; prop. 13: *nam sunt mutekefia*.

<sup>30</sup> Florence, BN, cs. J. I. 32, from the end of the thirteenth century.

<sup>31</sup> Clagett 1984, p. 224, variant to lines 7-9.

<sup>32</sup> See Kunitzsch 1993.

<sup>33</sup> Clagett 1984, pp. 161-163.

this proposition with a somewhat different proof could be found in Savasorda's *Liber embadorum* and in Leonardo Fibonacci's *Practica geometria*.<sup>34</sup>

Jordanus cites in his *Liber philotegni* three other treatises: the *Liber de curvis superficiebus*, the *De similibus arcubus* and the *Liber de ysoperimetris*. The first and second of these are mentioned in proposition 29, where Jordanus proves that the ratio of arcs cut off by unequal chords in a circle is greater than the ratio of the chords, and the ratio of the segments of the circle cut off by the chords will be greater than the square of the ratio of the chords. The *Liber de curvis superficiebus* attributed to Johannes de Tinemue, although probably translated from the Greek rather than from the Arabic, circulated with Arabo-Latin translations. This well-known treatise, dealing mostly with the surfaces and volumes of cones, cylinders, and spheres, gave the Latin West access to Archimedean methods and results.<sup>35</sup>

The author of the *Liber de similibus arcubus* was Ahmad b. Yūsuf b. Ibrāhīm ibn al-Dāya, who lived in the second half of the ninth century in Egypt and was the son of a well-known historian of medicine and man of letters.<sup>36</sup> Besides his work on ratios and proportions (in Latin: *De proportione et proportionalitate*), the first part of which has a similar purpose to book V of Euclid's *Elements* and continues with a treatment of the transversal figure, he wrote a treatise on similar arcs, which was also translated into Latin in the twelfth century, probably by Gerard of Cremona, under the title *Liber de similibus arcubus*.<sup>37</sup> In this treatise Ahmad tried to prove that the assertion "similar arcs are also equal arcs" was wrong. He starts with propositions III.20 and 21 of Euclid's *Elements*, and his short treatise could be seen as a commentary on book III of the *Elements*. Jordanus mentions this treatise in his *Liber philotegni* not only in proposition 29, but also in prop. 32<sup>38</sup> and in prop. 36.<sup>39</sup>

One of the more interesting propositions of the *Liber philotegni* is prop. 5: "If in a right triangle a line is drawn from one of the remaining angles to the base, the ratio of the angle farther from the right angle to the angle closer to the right angle is less than the ratio of its base to the base of the other."<sup>40</sup> Instead of giving a proof, Jordanus refers – correctly – to the demonstration in the *Liber de ysoperimetris*. This treatise, which was well-known in the Middle Ages, was not translated from the Arabic, but directly from the Greek.<sup>41</sup>

It should be noticed that the same proof was available in the *Optics* of Euclid (which was translated from the Arabic by Gerard of Cremona and also directly from the Greek) and in Gerard's translation of Ptolemy's *Almagest*. Jordanus cites the *Liber de ysoperimetris* again in prop. 30, then referring to his prop. 5.<sup>42</sup>

Clagett suggested that at two other places Jordanus used sources from the Arabic which are not known to us. In prop. 37, the difference or distance from AC to DE is greater than the distance from DE to FG, for  $CE > EG$ , which lines he here calls the distances of the tangents<sup>43</sup> (see fig. 3).

<sup>34</sup> Clagett 1984, pp. 163f.

<sup>35</sup> Edited by Clagett 1964, pp. 450-507.

<sup>36</sup> See Sezgin 1974, pp. 288-290.

<sup>37</sup> Ed. Busard and van Koningsveld 1973.

<sup>38</sup> *per librum de similibus arcubus*.

<sup>39</sup> *per librum de similibus arcubus*.

<sup>40</sup> Translation from Clagett 1984.

<sup>41</sup> Edited by Busard 1980.

<sup>42</sup> Clagett 1984, p. 226, line 9.

<sup>43</sup> See Clagett 1984, p. 280.

This curious term and the unnamed author (here: "he") have not been satisfactorily explained. Clagett suggests:

*It perhaps refers rather to the author of some fragment translated from the Arabic which Jordanus used as his source for this proposition and in which the term distantia was used in this rather unusual sense.*<sup>44</sup>

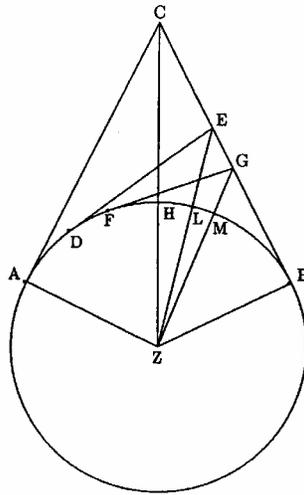


Figure 3.

Also strange is the form of prop. 40 whose enunciation gives a reason (*ratio*) on which it is based, but whose proof contains no reference to this and proceeds normally. Clagett comments:

*It seems obvious that Jordanus took this proposition (but not its proof) from some earlier work (perhaps a fragment translated from the Arabic).*<sup>45</sup>

In both cases it seems that Jordanus is quoting a work not known to us. Clagett's supposition of some fragment (or fragments) translated from the Arabic seems the most plausible explanation. Though some sources of the *Liber philotegni* are known, we do not know in general which parts were taken over and which are original contributions. But we agree with Clagett who writes:

*Regardless of how often Jordanus borrowed some proposition from treatises recently translated from the Arabic or the Greek, he put his own stamp on its demonstration, often producing an imaginative or ingenious proof. ...Jordanus... seemed to use the conventional theorems he inherited from his predecessors as an excuse for new ways of proving the old theorems or generating new ones. It is not surprising, then, that this work served as a magnet to attract other original and interesting propositions that circulated in translations from the Arabic but were not sufficiently*

<sup>44</sup> Clagett 1984, p. 280, note 2.

<sup>45</sup> Clagett 1984, p. 171.

*germane to Jordanus' objectives to have been included by him. The result of this attraction was the new version which we have called Liber de triangulis Iordani...<sup>46</sup>*

Clagett gives some arguments to prove that this *Liber de triangulis Iordani* was a revision of the *Liber philotegni* and was not written by Jordanus.<sup>47</sup> The most important is that in *De triangulis* the last 17 propositions of the *Liber philotegni* were omitted, which "are some of Jordanus' best propositions and ones that seem to represent the principal objectives of the *Liber philotegni*."<sup>48</sup> They were replaced by propositions IV.10, 12-28 of the *De triangulis*, most of which were parts of works translated from the Arabic, which were taken only with little changes, while Jordanus' skill lay in devising new proofs of his own. Two of Clagett's other arguments against the authorship of Jordanus are: in *De triangulis* Jordanus' name itself is cited, which would be unlikely if Jordanus himself were the author; and the mode of citing Euclid's *Elements* differs from that in the *Liber philotegni*.

The *De triangulis* did not use Campanus' redaction of Euclid's *Elements*. Therefore if it was written after this redaction – which was compiled at least no later than 1259 –, then its author probably did not have access to this text.<sup>49</sup> Apart from the *Liber de ysooperimetris*, the same treatises are cited as in the *Liber philotegni*. In addition, there are citations of Jordanus' *Arithmetica* and Ibn al-Haytham's *Perspectiva*.<sup>50</sup> The last work is mentioned in proposition IV.20, the classical problem of trisecting an angle. Other additions of Arabic origin are: construction of the regular heptagon and a proof of Hero's theorem.<sup>51</sup>

## 2.2. De elementis arismetice artis

Until 1991, when Busard edited this text from the original manuscripts, only the reworking by the French humanist Jacques Lefèvre d'Étaples (1496, reprinted 1514) was available. In this Jordanus' enunciations were given, but the demonstrations were not those of Jordanus, but by Lefèvre d'Étaples himself. In his edition Busard treats the question of Jordanus' sources. The following is based on remarks that Busard makes in his edition.<sup>52</sup>

It is surprising that in his *Arithmetica* Jordanus does not cite Euclid's *Elements* explicitly, although nearly all propositions of the arithmetical books of the *Elements*, i.e. books VII-IX, can be found in some way in the *Arithmetica*. Probably Jordanus used Version II of the *Elements*, because he arranged his book in a way similar to that version. Most striking is that in nearly all manuscripts of the *Arithmetica* the proofs precede the enunciations, and this is also true of the earlier manuscripts of the so-called Version II of the *Elements*. Another common feature is that in both texts very often not the full proofs are given, but only indications which propositions are necessary to develop the proof. Therefore Version II seems to have been the model for Jordanus in writing his *Arithmetica*.

Because Jordanus does not cite any author in his *Arithmetica* – except Boethius' *Arithmetica*<sup>53</sup> –, we are only able to list the propositions of Jordanus that can also be found in earlier texts and therefore might

<sup>46</sup> Clagett 1984, p. 185.

<sup>47</sup> Clagett 1984, pp. 297-303.

<sup>48</sup> Clagett 1984, p. 297.

<sup>49</sup> Clagett 1984, p. 301.

<sup>50</sup> Clagett 1984, p. 304.

<sup>51</sup> See Folkerts and Lorch 1992.

<sup>52</sup> Busard 1991, pp. 12-35.

<sup>53</sup> In VII.40: *quod Boecius dicit in Ysagogis arismetice*; see Busard 1991, p. 146.

have been taken from them. Apart from Version II, the most important are:<sup>54</sup> Al-Nayrīzi's commentary of Euclid and Ahmad b. Y´usuf's *De proportione et proportionalitate*, which were translated into Latin by Gerard of Cremona. Ahmad's treatise was the probable source for the definition of continued and discontinued proportion (II. def. 4, 5) and for several other passages. Unfortunately, we do not know the immediate source for Jordanus' very interesting treatment of the remainder problem (III.30, 31), but it is clear that it is ultimately based on Hindu mathematics. Nor is it known what sources, if any, Jordanus used for his solution of the indeterminate equation of the second degree:  $q^2 - v^2 = v^2 - r^2$  (VI.12), though problems of this sort were proposed and solved by Diophantus, al-Karaji, Leonardo Fibonacci and others.

### 2.3. De plana spera

Once again we find a text that is probably by Jordanus and expanded versions which probably are not.<sup>55</sup> In the text that the editor calls "Version 2", for instance, Euclid, or occasionally Theodosius, is frequently cited by proposition; and there are other signs of reworking. The text may be resolved into five propositions:

1. A demonstration that circles on the sphere become, when projected, circles on the plane;
2. and 3. On constructing parallels of given declination;
4. On the equal division of an oblique circle (although it is not so specified, we may consider this circle the ecliptic or the horizon);
5. On finding the position of a point whose coordinates with respect to a given oblique circle are known.

Much of the material is clearly based on corresponding passages in Ptolemy's *Planisphaerium*, which was translated into Latin in 1143 by Hermann de Carinthia from the Arabic. For the proof that circles become circles, which unfortunately appears to be not quite sound, no sources have yet been found – al-Farghānī, for instance, supplied a different proof, based on a proposition in Apollonius' *Conics*.<sup>56</sup> For Jordanus' fourth proposition, on the equal division of an oblique circle, three methods are given: by means of ascensions (if we may use the ordinary astronomical term), by declination circles, and by a special method involving the plane through the equinoxes which bisects the angle between the equator plane and the ecliptic plane. Circles perpendicular to this plane will cut off equal arcs from equator and ecliptic. Accordingly, such a circle is constructed for each division of the ecliptic (see fig. 4a) by constructing it through three points: the pole *K* of the circle, found as the intersection of the principal meridian *BED* and the line *AH* through one equinox *A* and through a point *H* on the equator circle distant from the other equinox by half the obliquity of the ecliptic, the point *L* on the equator whose distance from one of the equinoxes is equal to the desired arc of the ecliptic, and the point *M* opposite this point on the equator. The intersection of this circle with the ecliptic gives the desired division. The first two methods may be taken from Ptolemy's *Planisphaerium*. But all three methods are given in an extra chapter written by Maslama al-Majritī. The diagrams for the third are reproduced in fig. 4b. This chapter, which is extant in Arabic, was translated into Latin in the twelfth century and is almost certainly the ultimate source, if not the immediate source, of Jordanus' three methods.<sup>57</sup>

<sup>54</sup> See the list in Busard 1991, pp. 36-43.

<sup>55</sup> The three forms of the text have been edited with translation and commentary in Thomson 1978.

<sup>56</sup> See Thomson 1978, Appendix 3, pp. 210-217.

<sup>57</sup> The Arabic is in MS Paris, BN, arab. 4821, fols. 76r-79r, and was printed by Vernet and Catalá 1965. The Latin is in MS Vat. Reg. lat.

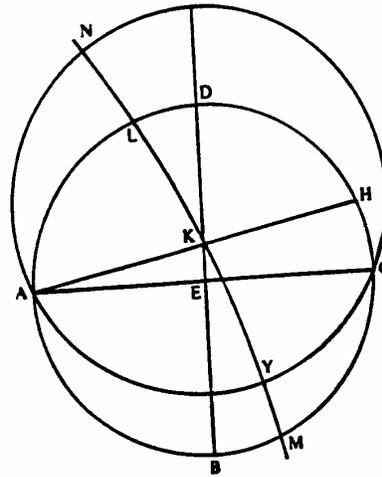


Figure 4a.

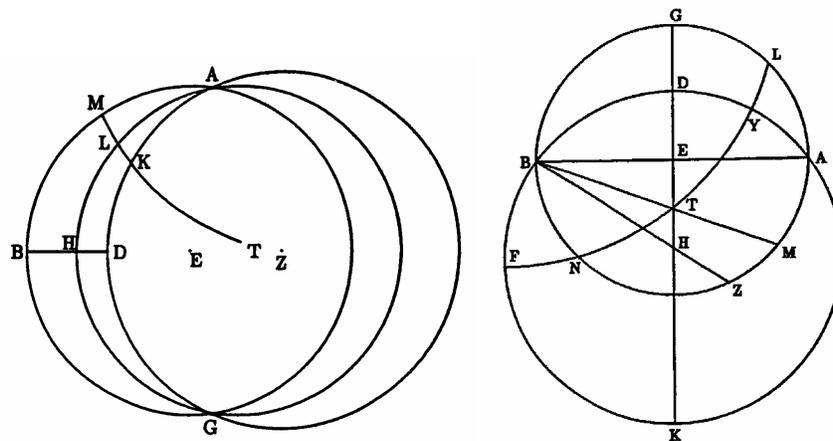


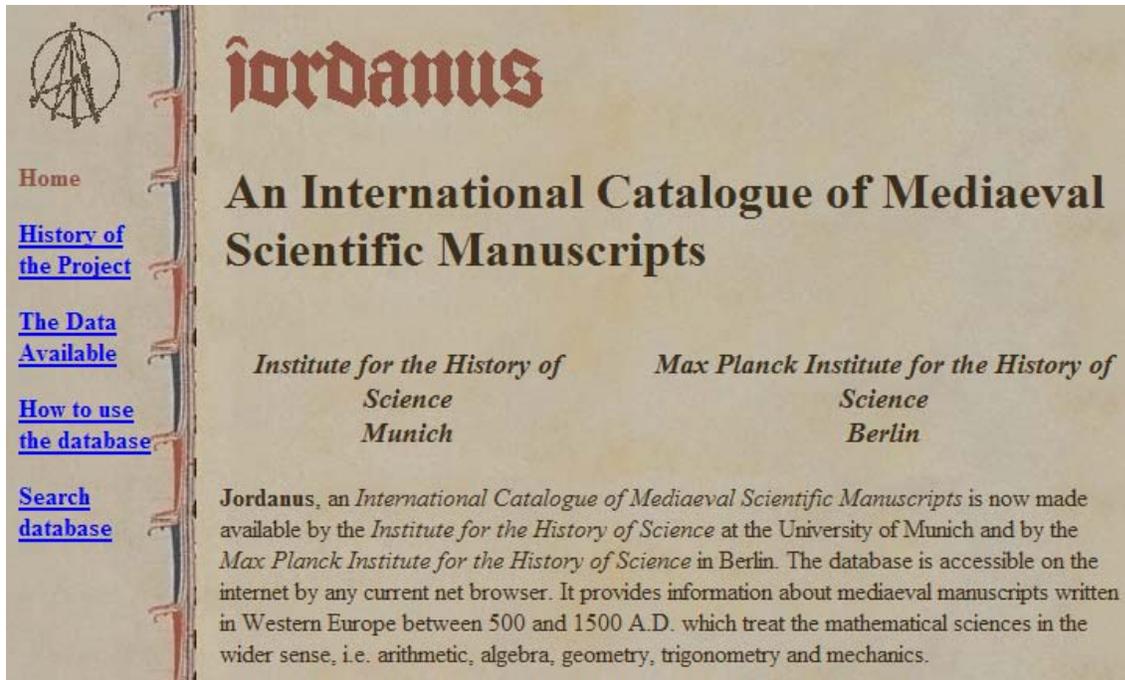
Figure 4b.

### 3. Final remarks

Jordanus was one of the few mathematicians of the Latin Middle Ages who showed any originality. He had also a strong inclination to rework the material that came to hand. None the less, it is possible to trace many of the ideas in his works to his predecessors, in particular to the translations from the Arabic in the twelfth century. All his major works were reworked, often more than once. It is remarkable that in many cases yet more material derived from the Arabic finds its way into the reworked texts, and this material is often more easily recognized, because it is more often supplied with the name of the source or because the style is less transformed. In the works by, or attributed to, Jordanus, which formed a large part of mathematics in the West from the founding of the universities until the Renaissance, we find a wonderful repository of mathematical learning transmitted from the rich Arabic heritage.

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1285, fols. 160va-162ra, and other manuscripts. The mathematical content is discussed in Anagnostakis 1984, chapter 11, pp. 171-178.



**Figure 5:** Homepage of *Jordanus: An International Catalogue of Mediaeval Scientific Manuscripts* (Joint project of the Institute for the History of Science (Munich) and the Max Planck Institute for the History of Science (Berlin)). Source: <http://jordanus.ign.uni-muenchen.de/cgi-bin/iccmsm>.

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